

Name: ANSWERSInstructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

In this quiz, the less shorthand the better. For example, when writing a formula for which you need a normal vector \vec{n} , don't just write " \vec{n} ", but rather the formula used to find it. Everything is positively oriented.

1. Define the following:

$$(a) \int_C f(x, y, z) ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt$$

$$(b) \int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

$$(c) \int_C f(x, y, z) dy = \int_a^b f(x(t), y(t), z(t)) y'(t) dt$$

(where C is a smooth curve parametrized by $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$. No shorthand, flesh out full definition.)

2. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$ 3. State the equation in Stokes' Theorem: $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl } \vec{F} \cdot d\vec{S}$ 4. What does it mean to say " \vec{F} is conservative"? $\vec{F} = \nabla f$ for some scalar function f .5. State the equation in Green's Theorem: $\int_C P dx + Q dy = \iint_D Q_x - P_y dA$ 6. State the equation in the Divergence Theorem: $\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \text{div } \vec{F} dV$ 7. Let $\vec{F} = \langle P(x, y), Q(x, y) \rangle$ be defined on an open, simply connected domain D . Suppose P and Q have continuous first partial derivatives on D . What equation would you use to check if \vec{F} is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ 8. Let $\vec{F} = \langle P(x, y), Q(x, y), R(x, y) \rangle$ be defined on an open, simply connected domain D . Suppose P , Q , and R have continuous first partial derivatives on D . What equation would you use to check if \vec{F} is conservative? $\text{curl } \vec{F} = \vec{0}$ 9. Let S_1 be a surface parametrized by $\vec{r}(s, t)$. Find a formula for a normal vector \vec{n}_1 to S_1 : $\vec{n}_1 = \pm \vec{r}_s \times \vec{r}_t$ 10. Let S_2 be a surface given by $z = g(x, y)$. Find a formula for a normal vector \vec{n}_2 to S_2 : $\vec{n}_2 = \pm \langle -g_x, -g_y, 1 \rangle$ 11. For S_1 above, define $\iint_{S_1} \vec{F}(x, y, z) \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(s, t)) \cdot (\vec{r}_s \times \vec{r}_t) dA$ 12. For S_2 above, define $\iint_{S_2} \vec{F}(x, y, z) \cdot d\vec{S} = \iint_D \vec{F}(x, y, g(x, y)) \cdot \langle -g_x, -g_y, 1 \rangle dA$