

Math 392 Quiz 6B

March 6, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x, y) dy = \int_a^b f(x(t), y(t)) y'(t) dt$

(b) $\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$

(c) $\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) \sqrt{(x'(t))^2 + (y'(t))^2} dt$

(where C is a smooth curve parametrized by $\vec{r}(t) = \langle x(t), y(t) \rangle$. No shorthand, flesh out full definition.)

2. What does it mean to say " \vec{F} is conservative"? $\vec{F} = \nabla f$ for some scalar function f .

3. State the equation in Green's Theorem: $\int_C P dx + Q dy = \iint_D Q_x - P_y dA$

4. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

5. Let $\vec{F} = \langle P(x, y), Q(x, y) \rangle$ be defined on an open, simply connected domain D . Suppose P and Q have continuous first partial derivatives on D . What equation would you use to check if \vec{F} is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

6. Let D be the triangle in the plane with vertices at $(0,0)$, $(2,0)$, and $(0,1)$. Let C be the positively oriented boundary of D .

Set-up integrals to compute (where a sum of integrals may be necessary): $\int_C 2xy dx + \left(\sin y + \frac{x^2 + y^2}{2} \right) dy$

(a) Line integral(s): $\int_0^1 -4t(2-2t) + \sin t + \frac{(2-2t)^2 + t^2}{2} - \sin(1-t) - \frac{(1-t)^2}{2} dt$

(b) Double integral(s): $\int_0^2 \int_0^{1-x/2} (-x) dy dx$

(c) Compute one of the parts above to give the value of the integral in 6. Ans: $-2/3$

Bonus:

1. Let $\vec{F} = \langle y^2, e^x, xyz^2 \rangle$, compute:

(a) $\text{curl } \vec{F} = \langle xz^2, -yz^2, e^x - 2y \rangle$

(b) $\text{div } \vec{F} = 2xyz$

2. If $\text{curl } \vec{F} = \vec{0}$, then \vec{F} is called irrotational

3. If $\text{div } \vec{F} = \vec{0}$, then \vec{F} is called incompressible