Math 392 Quiz 6A

July 16, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

In this quiz, the less shorthand the better. For example, when writing a formula for which you need a normal vector $\vec{n}$, don't just write "$\vec{n}$", but rather the formula used to find it. Everything is positively oriented.

1. Define the following:

(a) $\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

(b) $\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

(c) $\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt$

(where C is a smooth curve parametrized by $\vec{r}(t) = < x(t), y(t) >$. No shorthand, flesh out full definition.)

2. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

3. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

4. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f

5. State the equation in Stokes' Theorem: $\int_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{curl}\vec{F}\cdot d\vec{S}$

6. State the equation in the Divergence Theorem: $\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}\vec{F}\,dV$

7. Let $\vec{F} = < P(x,y), Q(x,y) >$ be defined on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

8. Let $\vec{F} = < P(x,y), Q(x,y), R(x,y) >$ be defined on an open, simply connected domain D. Suppose P, Q, and R have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\operatorname{curl}\vec{F} = \vec{0}$

9. Let S_1 be a surface given by $z = g(x,y)$. Find a formula for a normal vector $\vec{n}_1$ to S_1: $\vec{n}_1 = \pm < -g_x, -g_y, 1 >$

10. Let S_2 be a surface parametrized by $\vec{r}(s,t)$. Find a formula for a normal vector $\vec{n}_2$ to S_2: $\vec{n}_2 = \pm\, \vec{r}_s \times \vec{r}_t$

11. For S_1 above, define $\iint_{S_1} \vec{F}(x,y,z)\cdot d\vec{S} = \iint_D \vec{F}(x,y,g(x,y))\cdot < -g_x, -g_y, 1 >\,dA$

12. For S_2 above, define $\iint_{S_2} \vec{F}(x,y,z)\cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(s,t))\cdot(\vec{r}_s\times\vec{r}_t)\,dA$