Math 392 Quiz 6A

March 6, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x,y)\,ds =$ $\int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

(b) $\int_C \vec{F}\cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

(c) $\int_C f(x,y)\,dx =$ $\int_a^b f(x(t),y(t))\,x'(t)\,dt$

(where C is a smooth curve parametrized by $\vec{r}(t) =< x(t), y(t) >$. No shorthand, flesh out full definition.)

2. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

3. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

4. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f.

5. Let $\vec{F} =< P(x,y). Q(x,y) >$ be defined on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

6. Let D be the triangle in the plane with vertices at (0,0), (1,0), and (0,2). Let C be the positively oriented boundary of D.

Set-up integrals to compute (where a sum of integrals may be necessary): $\int_C \left(\cos x + \frac{x^2+y^2}{2}\right)dx + 2xy\,dy$

(a) Line integral(s): $\int_0^1 \cos t + \frac{t^2}{2} - \cos(1-t) - \frac{(1-t)^2+4t^2}{2} + 8t(1-t)\,dt$

(b) Double integral(s): $\int_0^1\int_0^{2-2x} y\,dy\,dx$

(c) Compute one of the parts above to give the value of the integral in 6. Ans: $2/3$

Bonus:

1. Let $\vec{F} =< x^2\,,\ e^y,\ xyz >$, compute:

(a) $curl\vec{F} =$ $\langle xz, -yz, 0\rangle$

(b) $div\,\vec{F} =$ $2x + e^y + xy$

2. If $curl\vec{F} = \vec{0}$, then $\vec{F}$ is called irrotational

3. If $div\,\vec{F} = \vec{0}$, then $\vec{F}$ is called incompressible