10

Math 392 Quiz 5B

March 6, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Suppose $\vec{F} = < P(x,y), Q(x,y) >$, define div$\vec{F}$ = $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = P_x + Q_y$

2. Find the curl and divergence of $\vec{F} = < x^3 - yz,\ y^3 - xz,\ z^3 - xy >$.

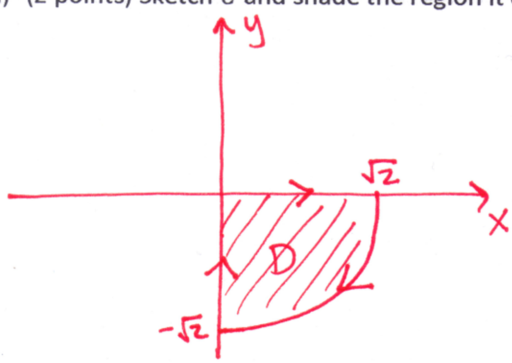
   (a) curl$\vec{F}$ = $\langle 0,0,0 \rangle = \vec{0}$

   (b) div$\vec{F}$ = $3x^2 + 3y^2 + 3z^2$ or $3(x^2+y^2+z^2)$

3. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$

4. Using Green's theorem, compute $I = \oint_C -2y\,dx + x\,dy$, where C is the boundary of the quarter-disc $x^2 + y^2 \leq 2$ in the fourth quadrant, oriented *clockwise* by doing the following:

   (a) (2 points) Sketch C and shade the region it encloses.



   (b) (2 points) Set up a double integral to compute I. I = $-\iint_D 3\,dA = -\int_{-\frac{\pi}{2}}^{0}\int_0^{\sqrt{2}} 3\,r\,dr\,d\theta$

   (c) (2points) Evaluate the integral set up in part (b): I = $-3\pi/2$

Bonus: A surface is parametrized by $\vec{r}(s,t) = < x(s,t),\ y(s,t),\ z(s,t) >$. Give a formula to compute a normal vector to the surface at any point (u,v).

$\vec{n}$ = $\vec{r}_s \times \vec{r}_t$

An alternative answer to problem 3: $\int_C \vec{F}\cdot d\vec{r} = \iint_D (Q_x - P_y)\,dA$