Math 392 Quiz 5B
July 9, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

In this quiz, the less shorthand the better. For example, when writing a formula for which you need a normal vector $\vec{n}$, don't just write "$\vec{n}$", but rather the formula used to find it. Everything is positively oriented.

1. Let S_1 be a surface parametrized by $\vec{r}(u,v)$. Find a formula for a normal vector $\vec{n}_1$ to S_1: $\vec{n}_1 =$ $\pm \vec{r}_u \times \vec{r}_v$

2. Let S_2 be a surface given by $z = g(x,y)$. Find a formula for a normal vector $\vec{n}_2$ to S_2: $\vec{n}_2 =$ $\pm \langle -g_x, -g_y, 1 \rangle$

3. For S_1 above, define $\iint_{S_1} \vec{F}(x,y,z) \cdot d\vec{S} =$ $\iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v)\, dA$

4. For S_2 above, define $\iint_{S_2} \vec{F}(x,y,z) \cdot d\vec{S} =$ $\iint_D \vec{F}(x,y,g(x,y)) \cdot \langle -g_x, -g_y, 1 \rangle\, dA$

5. State the equation in the Divergence Theorem: $\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F}\, dV$

6. Describe what the symbols above are and how they relate to each other: E is a solid, S is the simple, closed boundary of E.

7. State the equation in Stokes' Theorem: $\int_C \vec{F} \cdot d\vec{r} = \iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$

8. Describe what the symbols above are and how they relate to each other: S is a smooth surface w/ boundary curve C. C is closed, piecewise smooth.

9. Let $\vec{F} = \langle xyz, y, z \rangle$. Let S be the part of $z = x^2 + y^2$ that is below $z = 9$. Let C be the boundary curve of S. Fully set-up two integrals to compute the work done by $\vec{F}$ in moving a particle around C counter-clockwise.

   (a) Line integral: $\int_0^{2\pi} -243 \cos t \sin^2 t + 9 \cos t \sin t\, dt$

   (b) Double integral: $\int_0^{2\pi} \int_0^3 -9r^2 \cos\theta\, dr\, d\theta$

10. Let $\vec{F} = \langle xyz, y, z \rangle$. Let E be the region bounded by $z = x^2 + y^2$ and $z = 9$. Let S be the boundary of E. Fully set-up two integrals to compute the flux out of the surface of E.

    (a) Surface integral: $\int_0^{2\pi} \int_0^3 9r + r^3(2r^3 \cos^2\theta \sin\theta - 1) + 2r^3 \sin^2\theta\, dr\, d\theta$

    (b) Triple integral: $\int_0^{2\pi} \int_0^3 \int_{r^2}^9 (rz \sin\theta + 2) r\, dz\, dr\, d\theta$