Math 392 Quiz 5A

February 25, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x,y)\, ds = \int_a^b f(x(t), y(t)) \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

(b) $\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

(c) $\int_C f(x,y)\, dx = \int_a^b f(x(t), y(t))\, x'(t)\, dt$

(where C is a smooth curve parametrized by $\vec{r}(t) =< x(t), y(t) >$. No shorthand, flesh out full definition.)

2. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

3. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f.

4. Let $\vec{F} =< P(x,y). Q(x,y) >$ be defined on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

5. (a) Assume $f(x,y,z) = x \sin y + x^2 z$. Compute $\nabla f = \langle \sin y + 2xz, x\cos y, x^2 \rangle$

(b) Let C be the piece-wise smooth curve defined by: The line segment from (1,0,3) to (1,0,0), followed by the circular arc from (1,0,0) to (-1,0,0), then followed by another line segment from (-1,0,0) to (-2, $\pi/2$, 1). Compute:

$\int_C \nabla f \cdot d\vec{r} = f(-2, \frac{\pi}{2}, 1) - f(1,0,3) = -1$

where $\vec{r}(t)$ is your parametrization of C.

Bonus:

1. The field $\vec{F} =< e^y + yz - 2\,,\ xe^y + xz\,,\ xy >$ has a (scalar) potential function $f(x,y,z)$. You do not need to verify this. Find the general form of this potential function.

$f(x,y,z) = xe^y + xyz - 2x + C$

2. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\, dA$

3. Describe what all the symbols mean in the equation above: C is piece-wise smooth, positively oriented, closed curve. D is region bounded by C. P, Q are functions of x,y with continuous partial derivatives on an open region containing D, that are defined there.