**Math 392 Quiz 4B**

June 25, 2019

Name: ANSWERS

**Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.**

1. If $\vec{F} = < P(x,y,z), Q(x,y,z), R(x,y,z) >$, define $div\vec{F} =$ $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

2. Let $S_1$ be a surface parametrized by $\vec{r}(u,v)$. Find a formula for a normal vector $\vec{n}_1$ to $S_1$: $\vec{n}_1 =$ $\pm \vec{r}_u \times \vec{r}_v$

3. Let $S_2$ be a surface given by $z = h(x,y)$. Find a formula for a normal vector $\vec{n}_2$ to $S_2$: $\vec{n}_2 =$ $\pm \langle -h_x, -h_y, 1 \rangle$

4. What is the formula to compute the area of $S_1$ over a region $R$. $A =$ $\iint_R |\vec{r}_u \times \vec{r}_v| \, dA$

5. What is the formula to compute the area of $S_2$ over a region $D$. $A =$ $\iint_D \sqrt{(h_x)^2 + (h_y)^2 + 1} \, dA$

6. Let $\vec{F} = < -x^2, 0, 2xz - \cos x >$.

   (a) Compute $div\,\vec{F} =$ $0$

   (b) Does $\vec{F}$ have a vector potential? Yes (Yes/No)

   (c) If your answer above is "No", write "DNE" in the space provided. If "Yes", then find a vector potential $\vec{G}$ for $\vec{F}$. In doing so, you may assume the $z$-coordinate of $\vec{G}$ is 0, and set arbitrary constants of integration to 0 when convenient/appropriate.

   $\vec{G} =$ $\langle y\cos x, x^2 z, 0 \rangle$ or $\langle 0, x^2 z - \sin x, 0 \rangle$ other answers are possible!

7. Set-up integrals, with specific limits, to compute the areas of the following surfaces:

   (a) $\vec{r}(s,t) = < st, s+t, s-t >, 0 \le s,t \le 1$: $A =$ $\int_0^1 \int_0^1 \sqrt{4 + 2(s^2 + t^2)} \, ds\, dt$

   (b) The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$, and $x^2 + y^2 = 4$:

   $A =$ $\int_0^{2\pi} \int_1^2 \sqrt{4r^2 + 1} \, r\, dr\, d\theta$

**Bonus:**

1. For $S_1$ above, define $\iint_{S_1} f(x,y,z)dS =$ $\iint_R f(\vec{r}(u,v)) \, |\vec{r}_u \times \vec{r}_v| \, dA$

2. For $S_2$ above, define $\iint_{S_2} f(x,y,z)dS =$ $\iint_D f(x,y,h(x,y)) \sqrt{(h_x)^2 + (h_y)^2 + 1} \, dA$

(In this quiz, the less shorthand the better. Use as many variables as possible. For example, when writing a formula for which you need a normal vector $\vec{n}$, don't just write "$\vec{n}$", but rather the formula used to find it.)