Math 392 Quiz 4B
February 20, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x,y)\,dy =$ $\int_a^b f(x(t),y(t))\,y'(t)\,dt$

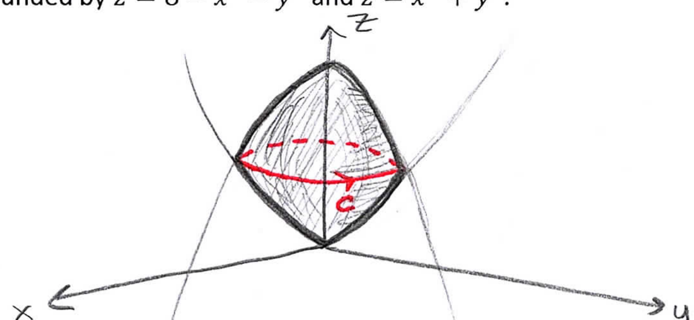
(b) $\int_C \vec{F}\cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

(c) $\int_C f(x,y)\,ds =$ $\int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

(where C is a smooth curve parametrized by $\vec{r}(t) = < x(t), y(t) >$. No shorthand, flesh out full definition.)

2. For us, what is the most important interpretation of $\int_C \vec{F}\cdot d\vec{r}$? Work! (By the vector field $\vec{F}$ in moving a particle along C.)

3. (a) Sketch the region bounded by $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.



(b) Parametrize the curve of intersection, C, of the above two surfaces. Set up the limits so that the curve is traversed once.

$C: \vec{r}(t) =$ $\langle 2\cos t, 2\sin t, 4\rangle$ Limits: $0 \le t \le 2\pi$

(c) Given $\vec{F} = < -y,\ x,\ x^2y^2 >$, find the work done by $\vec{F}$ in moving a particle around C once, by:

(i) Setting up an appropriate integral: Work $= \int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} 4\,dt$ (ii) Evaluating: 8π

(d) Set-up: $\int_C x^2y\,ds =$ $\int_0^{2\pi} 4\cos^2 t\cdot 2\sin t\cdot 2\,dt = \int_0^{2\pi} 16\cos^2 t\sin t\,dt$

Bonus:

1. What does it mean for $\vec{F}$ to be "conservative"? $\vec{F} = \nabla f$ for some scalar field f.

2. Suppose $\vec{F} = < P(x,y), Q(x,y) >$ and that P, Q, and their first order partial derivatives are continuous on $\mathbb{R}^2$.

What equation can be checked to see if $\vec{F}$ is conservative on $\mathbb{R}^2$? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$