Math 392 Quiz 4A

June 25, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let S_1 be a surface given by $z = g(x, y)$. Find a formula for a normal vector $\vec{n}_1$ to S_1: $\vec{n}_1 =$ $\pm\langle -g_x, -g_y, 1\rangle$

2. Let S_2 be a surface parametrized by $\vec{r}(s,t)$. Find a formula for a normal vector $\vec{n}_2$ to S_2: $\vec{n}_2 =$ $\pm\, \vec{r}_s \times \vec{r}_t$

3. What is the formula to compute the area of S_1 over a region D. $A =$ $\iint_D \sqrt{(g_x)^2+(g_y)^2+1}\, dA$

4. What is the formula to compute the area of S_2 over a region R. $A =$ $\iint_R |\vec{r}_s \times \vec{r}_t|\, dA$

5. If $\vec{F} = < P(x,y), Q(x,y) >$, define $div\vec{F} =$ $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$

6. Let $\vec{F} = < -y^2, 0, -x\cos y >$.

   (a) Compute $div\,\vec{F} =$ 0

   (b) Does $\vec{F}$ have a vector potential? Yes (Yes/No)

   (c) If your answer above is "No", write "DNE" in the space provided. If "Yes", then find a vector potential $\vec{G}$ for $\vec{F}$. In doing so, you may assume the z-coordinate of $\vec{G}$ is 0, and set arbitrary constants of integration to 0 when convenient/appropriate.

   $\vec{G} =$ $\langle x\sin y, y^2 z, 0\rangle$ or $\langle 0, y^2 z - \frac{x^2}{2}\cos y, 0\rangle$ other answers possible!

7. Set-up integrals, with specific limits, to compute the areas of the following surfaces:

   (a) $\vec{r}(u,v) = < uv, u+v, u-v >, u^2+v^2 \le 1$: $A =$ $\int_0^{2\pi}\int_0^1 \sqrt{4+2r^2}\, r\,dr\,d\theta$

   (b) The part of $y = 4x + z^2$ that lies between the planes $x = 0,\ x = 1, z = 0$, and $z = 1$:

   $A =$ $\int_0^1\int_0^1 \sqrt{17+4z^2}\, dx\,dz$

Bonus:

1. For S_1 above, define $\iint_{S_1} f(x,y,z)dS =$ $\iint_D f(x,y,g(x,y))\sqrt{(g_x)^2+(g_y)^2+1}\, dA$

2. For S_2 above, define $\iint_{S_2} f(x,y,z)dS =$ $\iint_R f(\vec{r}(s,t))\,|\vec{r}_s \times \vec{r}_t|\, dA$

(In this quiz, the less shorthand the better. Use as many variables as possible. For example, when writing a formula for which you need a normal vector $\vec{n}$, don't just write "$\vec{n}$", but rather the formula used to find it.)