Math 392 Quiz 4A

February 20, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

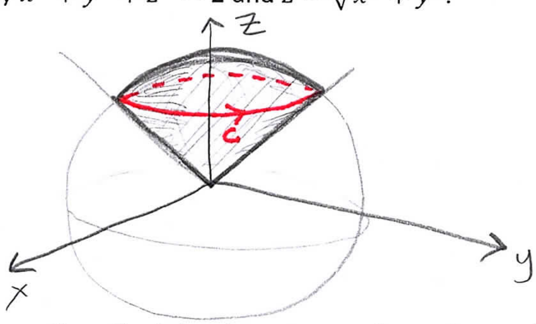
(b) $\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

(c) $\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt$

(where C is a smooth curve parametrized by $\vec{r}(t) = \langle x(t), y(t)\rangle$. No shorthand, flesh out full definition.)

2. For us, what is the most important interpretation of $\int_C \vec{F}\cdot d\vec{r}$? Work! (By the vector field $\vec{F}$ in moving a particle along C.)

3. (a) Sketch the region bounded by $x^2+y^2+z^2=2$ and $z=\sqrt{x^2+y^2}$.



(b) Parametrize the curve of intersection, C, of the above two surfaces. Set up the limits so that the curve is traversed once.

$C: \vec{r}(t) = \langle \cos t, \sin t, 1\rangle$ Limits: $0 \le t \le 2\pi$

(c) Given $\vec{F} = \langle -y,\ x,\ x^2y^2\rangle$, find the work done by $\vec{F}$ in moving a particle around C once, by:

(i) Setting up an appropriate integral: Work $= \int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} 1\,dt$ (ii) Evaluating: 2π

(d) Set-up: $\int_C x^3y\,ds = \int_0^{2\pi} \cos^3 t\,\sin t\,dt$

Bonus:

1. Suppose $\vec{F} = \langle P(x,y), Q(x,y)\rangle$ and that P, Q, and their first order partial derivatives are continuous on $\mathbb{R}^2$.

What equation can be checked to see if $\vec{F}$ is conservative on $\mathbb{R}^2$? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

2. What does it mean for $\vec{F}$ to be "conservative"? $\vec{F} = \nabla f$ for some scalar field f.