Math 392 Quiz 3B

February 11, 2019

Name: ANSWERS

**Instructions: No calculators! Answer <u>all</u> problems in the space provided! Do your rough work on scrap paper.**

1. Define the following:

   (a) $\nabla f(x,y) =$ $\langle f_x, f_y \rangle$

   (b) $\int_C f(x,y)\, ds =$ $\int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

   (where $C$ is a smooth curve parametrized by $\vec{r}(t) = < x(t), y(t) >$. No shorthand, flesh out full definition.)

2. Setup an integral to find the length of the curve parametrized by $x = 3e^t \cos t$, $y = 3e^t \sin t$ for $0 \le t \le 2\pi$.

   $L =$ $\int_0^{2\pi} 3\sqrt{2}\, e^t\, dt$ (Simplify the integrand, but do not evaluate the integral)

3. Evaluate the above integral: $L =$ $3\sqrt{2}(e^{2\pi}-1)$

   RECALL: $\sin 2x = 2\sin x \cos x$

4. Let $f = z\cos^2(xy)$, find $\nabla f =$ $\langle -yz\sin(2xy), -xz\sin(2xy), \cos^2 xy \rangle$

5. Let $C$ be the line segment from $(-1,-1)$ to $(1,1)$, compute $\int_C 2x^2\, ds$

   Integral set-up: $\int_0^1 4\sqrt{2}(-1+2t)^2\,dt$ Answer: $\frac{4\sqrt{2}}{3}$

**Bonus:**

1. Compute $\int_C 3y\, ds$ where $C$ consists of the quarter circle $x^2 + y^2 = 1$ in the second quadrant, traversed counter-clockwise, followed by the line segment from (-1,0) to (-2,0).

   Integral(s) set-up: $\int_{\pi/2}^{\pi} 3\sin t\, dt$ Answer: $3$

2. Define $\int_C \vec{F} \cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

   State the meanings of the symbols in the above: $\vec{F}$ is a vector field, $C$ is a smooth curve, $\vec{r}(t)$ is the parametrization of $C$.

   (Problem 2 is all-or-nothing)

3. Define $\int_C f(x,y)\, dy =$ $\int_a^b f(x(t),y(t))\,y'(t)\,dt$