Math 392 Quiz 3A

June 18, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define $\int_C \vec{F}\cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

2. Define $\int_C f(x,y)\,dx =$ $\int_a^b f(x(t),y(t))\,x'(t)\,dt$

3. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

4. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

5. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f

6. Let $\vec{F}$ be a vector field whose components have continuous first and second partials. What equation would you check to determine if $\vec{F}$ is conservative in the following cases?

(a) $\vec{F} = < P(x,y), Q(x,y) >$; equation to check: $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

(b) $\vec{F} = < P(x,y), Q(x,y), R(x,y) >$; equation to check: $\text{curl}\,\vec{F} = \vec{0}$

7. For us, what is the most important interpretation of $\int_C \vec{F}\cdot d\vec{r}$? Work

8. Find a scalar potential f for the function $\vec{F} =< \tan^{-1} y + z^2, \frac{x}{1+y^2}, 2xz >$. $f =$ $x\tan^{-1}y + xz^2 + C$

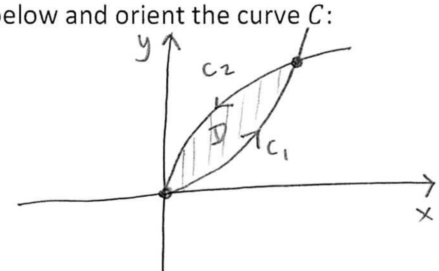
9. Let D be the region in the plane bounded by $x = y^2$ and $y = x^2$. Let C be the positively oriented boundary of D.

Set-up integrals to compute (where a sum of integrals may be necessary): $\int_C (xy + y^2)dx + (x - y)dy$

(a) Line integral(s): $\int_0^1 t^4 - t^3 + 2t^2\,dt + \int_0^1 t - t^2 - 2t^3 - 2t^4\,dt$

(b) Double integral(s): $\int_0^1\int_{x^2}^{\sqrt{x}} 1 - x - 2y\,dy\,dx$ OR $\int_0^1\int_{y^2}^{\sqrt{y}} 1 - x - 2y\,dx\,dy$

(c) Sketch the region below and orient the curve C:



Bonus:

1. What does it mean to say "$\vec{G}$ is a vector potential of $\vec{F}$"? $\text{curl}\,\vec{G} = \vec{F}$

2. Define $div\,\vec{F}(x,y,z) =$ $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$ where $\vec{F} = \langle P, Q, R\rangle$

3. If $curl\vec{F} = \vec{0}$, then $\vec{F}$ is called Irrotational ; if $div\,\vec{F} = 0$, then $\vec{F}$ is called incompressible