Math 392 Quiz 3A
February 11, 2019

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\nabla f(x, y, z) =$ $\langle f_x, f_y, f_z \rangle$

(b) $\int_C f(x, y)\, ds =$ $\int_a^b f(x(t), y(t)) \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

(where C is a smooth curve parametrized by $\vec{r}(t) = < x(t), y(t) >$. No shorthand, flesh out full definition.)

2. Setup an integral to find the length of the curve parametrized by $x = 2e^t \cos t$, $y = 2e^t \sin t$ for $0 \le t \le \pi$.

$L =$ $\int_0^{\pi} 2\sqrt{2}\, e^t\, dt$ (Simplify the integrand, but do not evaluate the integral)

3. Evaluate the above integral: $L =$ $2\sqrt{2}(e^{\pi} - 1)$

RECALL: $\sin 2x = 2 \sin x \cos x$

4. Let $f = z \sin^2(xy)$, find $\nabla f =$ $\langle yz \sin(2xy), xz \sin(2xy), \sin^2 xy \rangle$

5. Let C be the line segment from (1,1) to (2,2), compute $\int_C 3x^2\, ds$

Integral set-up: $\int_0^1 3\sqrt{2}(1+t)^2 dt$ Answer: $7\sqrt{2}$

Bonus:

1. Compute $\int_C 2x\, ds$ where C consists of the quarter circle $x^2 + y^2 = 1$ in the third quadrant, traversed counter-clockwise, followed by the line segment from (0,-1) to (0,-2).

Integral(s) set-up: $\int_{\pi}^{3\pi/2} 2 \cos t\, dt$ Answer: -2

2. Define $\int_C \vec{F} \cdot d\vec{r} =$ $\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$

State the meanings of the symbols in the above: $\vec{F}$ is a vector field, C is a smooth curve, $\vec{r}(t)$ is the parametrization of C.

(Problem 2 is all-or-nothing)

3. Define $\int_C f(x, y)\, dx =$ $\int_a^b f(x(t), y(t))\, x'(t)\, dt$