MATH 392 Quiz 2A

Name: ANSWERS

Instructions: No calculators! Use your own scrap paper and write your answers in the space provided.

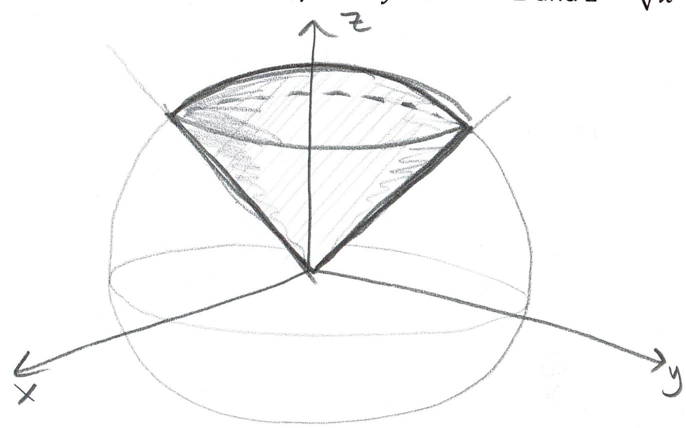
1. Let $\vec{r}(t) = < x(t), y(t) >$, $f(x,y)$ be a scalar function, and $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be points in $\mathbb{R}^3$. Complete the following rules with vector functions:

   (a) $\vec{r}\,'(t) = \langle x'(t), y'(t) \rangle$

   (b) $\nabla f = \langle f_x, f_y \rangle$

   (c) Line segment $\overrightarrow{PQ} = \langle x_1 + (x_2 - x_1)t, y_1 + (y_2 - y_1)t, z_1 + (z_2 - z_1)t \rangle, \ 0 \le t \le 1$ (include limits)

2. (a) (2 points) Sketch the region bounded by $x^2 + y^2 + z^2 = 2$ and $z = \sqrt{x^2 + y^2}$.



   (b) Parametrize the curve of intersection, $\vec{r}_i(t)$, of the above two surfaces. Set up the limits so that the curve is traversed once.

   $\vec{r}_i(t) = \langle \cos t, \sin t, 1 \rangle$ Limits: $0 \le t \le 2\pi$

3. (a) Parametrize the line segment from (1,-1,2) to (3,2,-1): $\vec{r}_l(t) = \langle 1+2t, -1+3t, 2-3t \rangle, 0 \le t \le 1$

   (b) What is the length of the above line? $L = \sqrt{22}$

4. Find a unit vector that is orthogonal to both $< 1,0,3 >$ and $< 2,-1,7 >$. $\vec{u} = \left\langle \frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}} \right\rangle$

   OR $\left\langle \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right\rangle$

Bonus:

1. Let $C = \vec{r}(t)$ and f be as in problem 1. Find formulas for:

   (i) The length of $\vec{r}(t)$ for $a \le t \le b$: $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

   (ii) $\int_C f\, ds = \int_a^b f(x(t), y(t)) \cdot \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

2. Compute the length of $\vec{r}(t) = < \cos^2 t, 4, \sin^2 t >$ for $0 \le t \le \frac{\pi}{2}$:

   Integral Set-up: $\int_0^{\pi/2} \sqrt{2} \sin 2t \, dt$, Answer: $\sqrt{2}$