MATH 392 TEST 1 REVIEW

October 5, 2011

From Spring 2005 Final:

1. Compute the vector which describes the direction of greatest increase for the function $f(x, y) = x^2y^3$ at the point with coordinates $(2, 1)$.

2. Find the equation of the tangent plane for the surface given by the equation $z = x^2y^3$ at the point with $(x, y) = (2, 1)$.

3. (a) Find the surface area of of the part of the paraboloid $z = 4 - x^2 - y^2$ contained in the first octant.
(b) Compute the directional derivative of the function $f(x, y) = x^2y^3$ in the direction from $(1, 2)$ to $(4, -2)$.

4. (a) For the vector field $\mathbf{F} = \langle ye^{xy} - z\sin xz, xe^{xy} + y^2, -x\sin xz\rangle$, compute a potential function $U(x, y, z)$ so that $\nabla U = \mathbf{F}$.
(b) Use your answer to part (a) to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ from $(0, 1, 0)$ to $(1, 2, \pi)$ along the path parametrized by $\langle t, t^2 + 1, 2\sin^{-1} t\rangle$ with $0 \le t \le 1$.

5. (a) For the path parametrized by $\mathbf{r}(t) = \langle t, \sin t, e^{2t}\rangle$, compute parametric equations for the tangent line at the point $(\pi, 0, e^{2\pi})$.
(b) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the path given in part (a) from $t = 0$ to $t = \pi$ where $\mathbf{F} = \langle y\cos x, y^2, z^2\rangle$.

6. Let R be the region $x + 2y \le 4$; $x \ge 0$; $y \ge 0$ in the $xy-$plane. Let C be the boundary of R, oriented counter-clockwise. Evaluate $\int_C (\sin x + y^2)\, dx + 2y\, dy$.

7. Let S be the portion of the plane $z = 2 - 2x - y$ which lies in the first octant. Let C be the boundary curve of S, oriented counter-clockwise as seen from above, and let $\vec{F} = \langle x, y, xyz\rangle$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

From Fall 2005 Final:

1. (a) Find the area of the part of the surface $z = xy - 1$ that lies inside the cylinder $x^2 + y^2 = 4$
(b) Find a function $f(x, y, z)$ with gradient $\nabla f = \langle 2xy + 2xz, x^2 + z, x^2 + y\rangle$

2. (a) Let S be the surface $z = \sqrt{x^2 + y^2}$ and let P be the point $(3, 4, 5)$ on S.
   (i)Find a normal vector to the surface S at the point P.
   (ii) Find an equation of the tangent plane to the surface S at the point P.

   (b) Let C be the curve parametrized by $x(t) = 3t + 1$, $y(t) = e^t$, $z(t) = 1$. Find parametric equations of the tangent line to the curve C at the point $(1, 1, 1)$.

3. Find the length of the parametrized curve given by the position vector $\vec{r}(t) = \left\langle \sqrt{2}, \cos^2 t, \sin^2 t \right\rangle$ with $0 \leq t \leq \dfrac{\pi}{2}$.

4. Let C be the boundary curve of the triangle with vertices $P(-1, 0)$, $Q(0, 1)$, and $R(1, 1)$, oriented counter-clockwise. Draw PQR and find $\int_C y^2 \, dx - x^2 \, dy$.

5. Let C be the curve of intersection of the cone $x^2 + y^2 = z^2$ and the plane $z = 3$, and let $\mathbf{F}$ be the vector field $y\mathbf{i} + z\mathbf{j} - x\mathbf{k}$. Let C be oriented clockwise as seen from above. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

From Spring 2006 Final:

1. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

2. Let $\mathbf{F} = \left\langle \dfrac{\ln y}{2\sqrt{x}} + yz, \dfrac{\sqrt{x}}{y} + xz, xy \right\rangle$.
   (a) Find a potential function $f(x, y, z)$ for $\mathbf{F}$ so that $\nabla f = \mathbf{F}$.
   (b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the straight line segment from $P(1, e, 1)$ to $Q(4, e^2, 2)$.

3. Find the length of the part of the parametrized curve $\vec{r}(t) = \left\langle \dfrac{t^2}{2}, \dfrac{2\sqrt{2}}{3} t^{3/2}, t \right\rangle$ between the points $P(0, 0, 0)$ and $Q\left(\dfrac{1}{2}, \dfrac{2\sqrt{2}}{3}, 1\right)$.

4. Let R be the region in the $xy-$plane bounded by the curves $x = y^2$ and $y = x - 2$. Find $\int_C -y \, dx + x \, dy$, where C is the boundary curve of R, oriented clockwise.

5. Let C be the curve of intersection of $z = x^2 + y^2$ and $z = 4$ with $y \geq 0$, oriented counter-clockwise as seen from above. Let $\vec{F} = < y, z, x >$. Find $\int_C \vec{F} \cdot d\vec{r}$.

From Fall 2006 Final:

1. Let $\vec{F}(x,y,z) = \langle ye^z + y, xe^z + x + 1, xye^z + 1\rangle$.
(a) Find a potential function $f(x,y,z)$ with $\nabla f = \vec{F}$.
(b) Let C be the straight line segment joining $(1,1,1)$ to $(2,2,0)$. Use the result of (a) to evaluate $\int_C \vec{F}\cdot d\vec{r}$.

2. (a) Find the surface area of the part of the surface $S:\ z = x^2 + y^2$ with $1 \le z \le 4$.
(b) Find the equation of the tangent plane to S at the point $(1,1,2)$.

3. Let R be the region bounded by the circle $x^2 + y^2 = 9$ and the lines $y = -x$ and $y = 0$ in the first and second quadrants. Suppose C is the boundary curve of R, oriented counter-clockwise. Evaluate $\int_C -y\ dx + x\ dy$.

4. Let S be the part of the surface $z = 1 - x^2$ in the first octant with $0 \le y \le 2$. Let C be the boundary curve of S, oriented counter-clockwise when viewed from above. If $\vec{F} = \left\langle 1, 0, y^2\right\rangle$, calculate $\int_C \vec{F}\cdot d\vec{r}$.

From Spring 2008 Final:

1. Evaluae the line integral $\int_C 6x^2\ ds$, where C is the part in the first quadrant of the circle of radius 4 centered at the origin.

2. Let $\vec{F} = \left\langle y^2e^{xy} + 6xy^2z, e^{xy} + xye^{xy} + 12z + 6x^2yz, 12y + 3x^2y^2\right\rangle$.
(a) Show that $\vec{F}$ is conservative; that is, find a scalar function f such that $\vec{F} = \nabla f$.
(b) Find the work done by $\vec{F}$ along the path from $(0,1,0)$ to $(3,4,-1)$ to $(1,-1,3)$ along two straight line segments.

3. Find the work done by the vector field $\vec{F} = \left\langle e^x + x^2y, e^y - xy^2\right\rangle$ around the circle of radius 3 centered at the origin travelled clockwise.

4. Let C be the curve of intersection of the surfaces $z = 3x - 7$ and $x^2 + y^2 = 1$, oriented clockwise as seen from above. Let $\vec{F} = \langle 4z - 1, 2x, 5y + 1\rangle$. Compute the work integral $\int_C \vec{F}\cdot d\vec{r}$.