MATH 391

1- For the equation $(x+y)dx+(x-y)dy=0$ consider the properties:
I The equation is separable.
II The equation is linear.
III The equation is homogeneous.
IV The equation is exact.
Which of the following are true
(a) only I.
(b) only II.
(c) only III.
(d) I and IV.
(e) III and IV.

2- For the equation $(x+y)dx+(y-x)dy=0$ consider the properties:
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3- The equation $y' = \frac{x(x^2+1)}{4y^3}$ is satisfied when
(a)$-2y^{-2} = \ln(x) \cdot \arctan(x) + C$.
(b)$-2y^{-2} = x^2 + \arctan(x) + C$.
(c) $y^4 = \ln(x(x^2+1)) + C$.
(d) $4y^4 = x^4 + 2x^2 + C$.
(e) None of the other alternatives is correct.

4- For the equation $y' = \frac{y(x+y)}{x^2}$ we use the substitution $z = y/x$ to obtain

(a)$x\frac{dz}{dx} = \frac{z(1+z)}{x^2}$.
(b)$x\frac{dz}{dx} + z = \frac{z(1+z)}{x^2}$.
(c)$x\frac{dz}{dx} = \frac{z(1+z)}{1}$.
(d) $x\frac{dz}{dx} = z^2$.
(e) None of the other alternatives is correct.

5- For the initial value problem
$e^x(\cos(xy) - y\sin(xy))dx + (y - xe^x\sin(x,y))dy = 0, \qquad y(0) = 5$ is
(a) The equation is exact and the solution is $y = e^x\cos(xy) + 4$.
(b) The equation is exact and the solution is $y = e^x\cos(xy) + y^2 - 21$.
(c) The equation is exact and the solution is $27 = 2e^x\cos(xy) + y^2$.
(d) The equation is not exact.

6- With an initial investment of \$100, I invest \$500 per year, continuously, at an interest rate of 5%, leaving the money in the bank. Assuming the interest is compounded continuously, the initial value problem which describes the amount $\$S$ in the bank t years later is given by:
(a) $S = (1.05)^t 500, \quad S(0) = 100$.
(b) $\frac{dS}{dt} = 500e^{.05t}, \quad S(0) = 100$.
(c) $\frac{dS}{dt} = \frac{S}{20} + 500, \quad S(0) = 100$.
(d) $\frac{dS}{dt} = .05S + 100, \quad S(0) = 500$.
(e) $S = e^{.05t}500, \quad S(0) = 100$.

7- For the second order ODE $t^2y'' + yy' = \sin(y)$ we apply the substitution $v = y'$ and obtain
(a) the first order ODE $t^2v' + yv = \sin(y)$.
(b) the first order ODE $y^2v' + yv = \sin(t)$.
(c) the first order ODE $t^2v\frac{dv}{dy} + yv = \sin(y)$.
(d) None of the alternatives is a first order ODE.

8- Assume that y_1 and y_2 are solutions of the equation $y'' + p(t)y' + q(t)y = 0$ with all functions defined on the entire real line. Let $W = W(y_1, y_2)$ be the Wronskian of the pair of functions. Which of the following statements are not true

I $\frac{dW}{dt} = y_1y_2'' - y_2y_1''$.
II $\frac{dW}{dt} = pW$.
III $\frac{dW}{dt} = qW$.
IV If $W(0) > 0$ then W is a positive constant.
(a) Just I and IV.
(b) Just III and IV.
(c) Just I and II.
(d) Just I and III.
(e) II, III and IV.

9- If y_1 is a solution of the equation $y'' + p(t)y' + q(t)y = 0$, then $y_2 = uy_1$ is also a solution when u satisfies the equation.
(a) $p\frac{du}{dt} + q = 0$.
(b) $p\frac{d^2u}{dt^2} + q\frac{du}{dt} = 0$.
(c) $y_1\frac{du}{dt} + (2y_1' + py_1)u = 0$.
(d) $y_1\frac{d^2u}{dt^2} + p\frac{du}{dt} = 0$.
(e) None of the other alternatives is correct.

10- For the equation $y'' - y' - 2y = 3t^{-1}e^{-t}$, the general solution consists of the general solution of the homogeneous equation added to a particular solution of the form:
(a) $At^{-1}e^{-t}$.
(b) $(At^{-1} + B)e^{-t}$.
(c) $u_1e^{-t} + u_2e^{2t}$ with $u_1'e^{-t} + u_2'e^{2t} = 3t^{-1}e^{-t}$.
(d) $u_1e^{-t}+u_2e^{2t}$ with $u_1e^{-t}+u_2e^{2t} = 0$ and $-u_1e^{-t}+2u_2e^{2t} = 3t^{-1}e^{-t}$.
(e) None of the other alternatives is correct.

11- For the equation $y'' - 4y' + 4y = \tan(2t)$, the general solution consists of the general solution of the homogeneous equation added to a particular solution of the form:
(a) $A\tan(2t)$.
(b) $(A + Bt)\tan(2t)$.
(c) $u_1e^{2t} + u_2te^{2t}$ with $u_1'e^{2t} + u_2'te^{2t} = \tan(2t)$.
(d) $u_1e^{2t} + u_2te^{2t}$ with $u_1'2e^{2t} + u_2'(1 + 2t)e^{2t} = \tan(2t)$ and $u_1'e^{2t} + u_2'te^{2t} = 0$.
(e) None of the other alternatives is correct.

12- For the equation $y^{(5)} - y'' = 7t^2 + e^t - \cos(\frac{\sqrt{3}}{2}t) + 5$, the general solution consists of y_h , general solution of the homogeneous given by (1), added to y_p, a particular solution obtained from the method of undetermined coefficients whose simplest form is given by (2):

(a) (1) $y_h = C_1 + C_2 e^t$, and (2) $y_p = At^2 + B + C\cos(\frac{\sqrt{3}}{2}t)$.

(b) (1) $y_h = C_1 + C_2 t + C_3 e^t + C_4 t e^t + C_5 t^2 e^t$ and (2) $y_p = At^2 + Bt + C + De^t + E\cos(\frac{\sqrt{3}}{2}t) + F\sin(\frac{\sqrt{3}}{2}t)$.

(c) (1) $y_h = C_1 + C_2 t + C_3 e^t + C_4 t e^t + C_5 t^2 e^t$ and (2) $y_p = At^2 + Bt + C + Dt^2 e^t + E\cos(\frac{\sqrt{3}}{2}t) + F\sin(\frac{\sqrt{3}}{2}t)$.

(d) (1) $y_h = C_1 + C_2 t + C_3 e^t + C_4 t e^t + C_5 t^2 e^t$ and (2) $y_p = At^2 + Bt + C + Dt^2 e^t + Ete^t + Fe^t + G\cos(\frac{\sqrt{3}}{2}t) + H\sin(\frac{\sqrt{3}}{2}t)$.

(e) None of the other alternatives is correct.

13- For the equation $(3x^2 + 2)y'' - 7x^3 y = 0$ we look for a series solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$. The recurrence formula for the coefficients is given by

(a) $a_{k+3} = \frac{1}{7}[2a_{k+2} + 3a_k]$.

(b) $a_{k+3} = \frac{1}{7}[2(k+2)(k+1)a_{k+2} + 3k(k-1)a_k]$.

(c) $a_{k+2} = \frac{1}{2(k+2)(k+1)}[7a_{k-3} - 3k(k-1)a_k]$.

(d) $a_{k+2} = \frac{1}{3(k+2)(k+1)}[7a_{k-3} - 2k(k-1)a_k]$.

(e) None of the other alternatives is correct.

14- For the equation $x^2 y'' + 7xy' + 9y = 0$, the general solution is

(a) $y = C_1 x^3 + C_2 x^3 \ln(x)$

(b) $y = C_1 e^{3t} + C_2 e^{3t} \ln(t)$.

(c) $y = C_1 x^{(-7+\sqrt{13})/2} + C_2 x^{(-7-\sqrt{13})/2}$.

(d) $y = C_1 x^{-7/2}\cos(\sqrt{13}x/2) + C_2 x^{-7/2}\sin(\sqrt{13}x/2)$.

(e) None of the other alternatives is correct.

15- For the equation $4x^2 y'' - x^2 y' + (1+x)y = 0$ we look for a series solution of the form $y = x^r \sum_{n=0}^{\infty} a_n x^n$ with r the smallest root of the indicial equation. The recurrence formula for the coefficients is given by

(a) $a_k = \frac{2k-3}{8k^2} a_{k-1}$.

(b) $a_k = \frac{4k-7}{8k^2} a_{k-1}$.

(c) $a_k = \frac{4}{(4k-1)^2}(k - 2 + \frac{1}{4})a_{k-1}$.

(d) $a_{k-\frac{1}{2}} = \frac{1}{4(k+2)(k+1)}(3ka_{k-\frac{3}{2}} - a_{k-1})$.

(e) None of the other alternatives is correct.

16- Which of the following can be a solution of the heat equation $u_t = 4u_{xx}$ on the interval $x \in [0,1]$ with the homogeneous boundary conditions $u(t,0) = u(t,1) = 0$ for all $t > 0$.

I $u = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-(n\pi/2)^2 t} \cos(n\pi x)$.

II $u = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2} e^{(2n\pi)^2 t} \cos(n\pi x)$.

III $u = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-(2n\pi)^2 t} \sin(n\pi x)$.

IV $u = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{(n\pi/2)^2 t} \sin(n\pi x)$.

(a) Only I.

(b) Only I or II.

(c) Only III.

(d) Only I or III.

(e) Only III or IV.