Math 346 Quiz 3B
February 17, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Recall: if $A = [a_{ij}]_{n\times m}$, then $A^T = [a_{ji}]_{m\times n}$. Suppose $B = [b_{ij}]_{n\times m}$. Prove that $(A+B)^T = A^T + B^T$.

Pf: $(A+B)^T = [a_{ij}+b_{ij}]^T = [a_{ji}+b_{ji}] = [a_{ji}]+[b_{ji}] = A^T + B^T$. $\blacksquare$

2. Let $A = [a_{ij}]_{n\times n}$. Define $tr(A) =$ $a_{11}+a_{22}+\dots+a_{nn}$ or $\sum_{i=1}^{n} a_{ii}$

3. (a) Given $A = [a_{ij}]_{n\times n}$ and $B = [b_{ij}]_{n\times n}$, conjecture a formula for $tr(A+B) =$ $tr(A)+tr(B)$
(b) Prove your formula works:

Pf: $tr(A+B) = \sum_{i=1}^{n}(a_{ii}+b_{ii}) = \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} b_{ii} = tr(A)+tr(B)$. $\blacksquare$

4. Solve the system
$$\begin{aligned} x + 2y - z &= -2 \\ x + z &= 2 \\ 2x - 4y + z &= 7 \end{aligned}$$
by doing the following:

(a) Write down the augmented matrix for the system:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 2 \\ 2 & -4 & 1 & 7 \end{array}\right)$$

(b) Find the reduced row-echelon form of the augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 2 \\ 2 & -4 & 1 & 7 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & -2 & -4 \\ 0 & 8 & -3 & -11 \end{array}\right) \begin{array}{l} R_2 \\ R_1 - R_2 \\ 2R_1 - R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -5 & -5 \end{array}\right) \begin{array}{l} R_1 \\ R_2/2 \\ 4R_2 - R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \begin{array}{l} R_3/5 + R_1 \\ R_3/{-5} + R_2 \\ R_3/{-5} \end{array}$$

(c) Write down the solution as a column vector: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Bonus: (a) If $A = \begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix}$, find $A^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1/2 & 1 \end{pmatrix}$

(b) If $B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$, find $\det B =$ 2

(c) Is it possible to find B^{-1}? Yes How do you know? $\det B \neq 0$

(d) Let C be the coefficient matrix of the system in problem 4. Is it possible to find C^{-1}? Yes

How do you know? RREF of $C = I_3$