Math 346 Quiz 2B

February 8, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. For the system:
   $a_1x + a_2y = b_1$
   $a_3x + a_4y = b_2$
   Assuming $a_1a_4 - a_2a_3 \neq 0$, write down a formula for $x = \dfrac{\begin{vmatrix} b_1 & a_2 \\ b_2 & a_4 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \dfrac{a_4b_1 - a_2b_2}{a_1a_4 - a_2a_3}$, $y = \dfrac{\begin{vmatrix} a_1 & b_1 \\ a_3 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \dfrac{a_1b_2 - a_3b_1}{a_1a_4 - a_2a_3}$

2. For the matrix $A = [a_{ij}] = \begin{pmatrix} 7 & 2 & 3 \\ 5 & 0 & -1 \\ 6 & 7 & \pi \end{pmatrix}$, what is $a_{32} =$ 7 ?

3. Let $A = \begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 9 \\ -1 & 1 & 5 \\ 3 & 4 & 7 \end{pmatrix}$, $C = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Compute the following, or write "DNE", for "does not exist".

   (d) $B + 2D =$ DNE

   (b) $AB = \begin{pmatrix} 9 & 6 & 13 \\ 6 & 4 & 34 \end{pmatrix}$

   (c) $BA =$ DNE

   (d) $A - 3D = \begin{pmatrix} -2 & -2 & 2 \\ 3 & 0 & -2 \end{pmatrix}$

4. Suppose A and C above were multiplied to find CA. Write the size of the result, or "DNE" if they actually cannot be multiplied: 2×3

5. List the square matrices in problem 3. B, C

6. True or false: Suppose AB is defined. If B has a column of zeros, then AB has a column of zeros. True

7. Justify your answer in problem 6.
   Suppose $A = [A]_{n\times m}$ and $B = [B]_{m\times p}$. Assume $\vec{b}_j = \vec{0}_m$ is the j^{th} column of B. Then the j^{th} column of AB would be $[\vec{a}_{(1)}\cdot\vec{b}_j \; \vec{a}_{(2)}\cdot\vec{b}_j \; \cdots \; \vec{a}_{(n)}\cdot\vec{b}_j]^T = [0\,0\ldots 0]^T$. So that AB has a column of zeros; namely, its j^{th} column. ▨

8. Would your answer to problem 6 change if it were A that had the column of zeros? Yes

Bonus: (a) What is $tr(C) =$ $2 + 0 = 2$?

(b) Write the system in problem 1 as an augmented matrix below:

$$\left(\begin{array}{cc|c} a_1 & a_2 & b_1 \\ a_3 & a_4 & b_2 \end{array}\right)$$

(c) Justify your answer to problem 8.
If A had a column of zeros, the statement would be false.
Counter example: $A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow AB = \begin{pmatrix} 3 & 4 \\ 3 & 4 \end{pmatrix}$. (many examples possible).
So A has a column of zeros, but AB does not. □