

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. For the system:

$$ax_1 + by_1 = c$$

$$dx_1 + ey_1 = f$$

Assuming $ae - bd \neq 0$, write down a formula for $x_1 =$

$$\frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} = \frac{ce - bf}{ae - bd}, y_1 = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} = \frac{af - cd}{ae - bd}$$

2. For the matrix $A = [a_{ij}] = \begin{pmatrix} 7 & 2 & 3 \\ 5 & 0 & -1 \\ 6 & 7 & \pi \end{pmatrix}$, what is $a_{13} =$ 3?

3. Let $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 7 \\ 1 & -1 & 5 \\ 3 & 4 & 9 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$. Compute the following, or write "DNE", for "does not exist".

(a) $A + 2D = \begin{pmatrix} 3 & 0 & 3 \\ 2 & 1 & 1 \end{pmatrix}$ (b) $AB = \begin{pmatrix} 10 & 12 & 34 \\ 4 & 5 & 18 \end{pmatrix}$

(c) $BA =$ DNE (d) $B - 3A =$ DNE

4. Suppose C and D above were multiplied to find CD . Write the size of the result, or "DNE" if they actually cannot be multiplied: 2×3

5. List the square matrices in problem 3. B, C

6. True or false: Suppose AB is defined. If A has a row of zeros, then AB has a row of zeros. True

7. Justify your answer in problem 6.

Suppose $A = [A]_{n \times m}$ and $B = [B]_{m \times p}$. Assume $\vec{a}_{(i)} = \vec{0}_m$ is the i^{th} row of A . Then the i^{th} row of AB would be $[\vec{a}_{(i)} \cdot \vec{b}_1, \vec{a}_{(i)} \cdot \vec{b}_2, \dots, \vec{a}_{(i)} \cdot \vec{b}_p] = [0, \dots, 0]$. So that AB has a row of zeros; namely, its i^{th} row. \square

8. Would your answer to problem 6 change if it were B that had the row of zeros? Yes

Bonus: (a) What is $\text{tr}(B) =$ $1 + (-1) + 9 = 9$?

(b) Write the system in problem 1 as an augmented matrix below:

$$\left(\begin{array}{cc|c} a & b & c \\ d & e & f \end{array} \right)$$

(c) Justify your answer to problem 8.

If B had a row of zeros, the statement would be false. Counter example: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \Rightarrow AB = \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}$ (many examples possible). So B has a row of zeros, but AB does not. \square