City College **Department of Mathematics**

Math 346 **Sample Final Exam Problems**

1. Consider the following augmented matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{pmatrix}$$

Which statements are true? (Mark all that apply)

(a) The system is inconsistent

(b) The system has infinitely many solutions

(c) There are no free variables

(d) The system has a unique solution.

2. Let A and B be $n \times n$ matrices such that $AB = I$, where I is the $n \times n$ identity matrix. Mark all statements that are necessarily correct:

(a) $BA = I$

(b) $BA^{-1} = I$

(c) $AB = BA$

(d) $det(A) = det(B)$

(e) There is a (column) vector $C = (c_1, \ldots, c_n)$ such that the system $AX = C$ has infinitely many solutions.

3. Let

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 1 & -1 & t \\ 2 & -2 & -2t+1 \end{pmatrix}.$$

Suppose the third column of A^{-1} is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Then $a + 2b$ is equal to

(a) 2

(b) -2

(c) 1

(d) -1

4. Suppose a 2×2 matrix A has eigenvalues 3 and 5. What is the trace of the matrix $A^2 + 2A$?

(a) 45

(b) 63

(c) 50

(d) 320

5. Let W be the set of all vectors of the form

$$\begin{pmatrix} a - 4b \\ 2 \\ 6a + b \\ -a - b \end{pmatrix}.$$

Then (mark all that apply):

(a) $\begin{pmatrix} 1 \\ 2 \\ 6 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \\ -1 \end{pmatrix}$ is a basis of W.

(b) $\begin{pmatrix} 1 \\ 0 \\ 6 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ 1 \\ -1 \end{pmatrix}$ is a basis of W.

(c) W is not a vector space.

(d) $\begin{pmatrix} 1 \\ 0 \\ 6 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ is a basis of W.

6. Suppose B is a 2×2 matrix such that $det(B) = -2$. What is the determinant of the matrix $A = 3B^3(B^T)^2B^{-1}$?

(a) -48

(b) 144

(c) 48

(d) 120

7. Let A be a 3×3 matrix with determinant 10. We add to the third column of A the second column multiplied by 3, then multiply the first column by (-1), and then we switch the second and third rows. Denote the resulting matrix by B. What is the determinant of the matrix $2B^2(B^T)^{-1}$?

(a) 36

(b) -16

(c) 80

(d) 120

(e) 150

8. The line which best fits the points (0, 1), (1,2), (-1,-1), and (2,0) is given by:

(a) $y = \frac{1}{10} + \frac{2}{5}x$

(b) $y = \frac{3}{10} + \frac{2}{5}x$

(c) $y = \frac{3}{10} + 5x$

(d) $y = \frac{1}{10} + 5x$

(e) $y = \frac{1}{10} - \frac{2}{5}x$.

9. Let $a = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. The matrix P that projects every vector in R^3 onto the orthogonal component of the line spanned by the vector a is:

(a)

$$P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

(b)

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(c)

$$P = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

(d)

$$P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(e) None of the above.

10. Find $y_1(t)$ and $y_2(t)$, where

$$\begin{cases} y_1'(t) & = & y_1(t) & - & 2y_2(t) \\ y_2'(t) & = & -2y_1(t) & + & y_2(t) \end{cases}$$

and $y_1(0) = 1$ and $y_2(0) = 3$. What is $y_1(1)$?

(a) $\frac{3}{2}(e^{-1} - e^3)$,

(b) $\frac{3}{2}(e^{-1} + e^3)$, **(c)** $\frac{1}{2}(e^{-1} - e^3)$, **(d)** $\frac{1}{2}(e^{-1} + e^3)$, **(e)** None of the above.

11. Let $T : R^2 \to R^3$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ -1 & -3 \end{pmatrix}.$$

Which of the following vectors are not in the range of T? (Mark all that apply.)

(a) $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$, **(b)** $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, **(c)** $\begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$, **(d)** $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, **(e)** $\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$