Math 308 Quiz 8

June 24, 2015

Name: ANSWERS

Instructions: Answer all problems in the space provided. Show all work.

1. Prove that $1+5+9+\cdots+(4n-3)=2n^2-n$ for every $n\in\mathbb{N}$. (4 points)

Pf: By induction,

Base case: $n=1$, $1=2(1)^2-1=1$ ✓

Inductive step: Assume $1+\cdots+4n-3=2n^2-n$, we show $1+\cdots+4(n+1)-3=2(n+1)^2-(n+1)$.

Since $1+5+\cdots+4n-3=2n^2-n$

$$\Rightarrow 1+5+\cdots+4n-3+4(n+1)-3 = 2n^2-n+4(n+1)-3$$
$$= 2n^2+4n+2-n-1$$
$$= 2(n+1)^2-(n+1).$$

▨

2. Prove that $n!>2^n$ for every integer $n\geq 4$. (4 points)

$P(n)$:

Pf: By induction,

Base case: $P(4)\Rightarrow n=4$: $4!=24>16=2^4$ ✓

Assume $P(n)$ holds. We show $P(n+1)$ holds.

$P(n)$ holds $\Rightarrow n!>2^n$

$$\Rightarrow (n+1)! > (n+1)2^n$$
$$= n\cdot 2^n + 2^n$$
$$> 2^n+2^n, \text{ since } n\geq 4$$
$$= 2\cdot 2^n$$
$$= 2^{n+1}$$

▨

3. Prove that $4|(5^n-1)$ for every nonnegative integer n. (5 points)

$P(n)$:

Pf: By induction,

Base case: $P(0)\Rightarrow n=0$: $4\,|\,[(5^0-1)=0]$ ✓

Assume $P(n)$ holds. We show $P(n+1)$ holds.

$P(n)$ holds $\Rightarrow 5^n-1=4k$ for $k\in\mathbb{Z}$ $\Rightarrow 5^n=4k+1$.

Then $5^{n+1}-1 = 5\cdot 5^n-1$
$$= 5(4k+1)-1$$
$$= 4(5k+1)$$

$$\Rightarrow 4\,|\,(5^{n+1}-1).$$

▨

Bonus problems:

1. Let $P(n)$ be a statement for $n\in\mathbb{N}$:

(a) Describe a proof of $P(n)$ by *minimum counterexample.*

Assume $P(n)$ is false for some $n\in\mathbb{N}$. There must be a minimum $n\in\mathbb{N}$ for which it's false. Call this m. Show $P(m)$ holds. ↯

(b) Describe the structure of the proof for $P(n)$ using the *Strong Principle of Mathematical induction.*

To show $P(n)$ holds for all $n\in\mathbb{N}$. Show

(1) $P(1)$ is true.

(2) $P(i)$ true for $1\leq i\leq k$, then $P(k+1)$ is true.

$\Rightarrow P(n)$ holds.