Math 308 Quiz 7

June 22, 2015

Name: ANSWERS

Instructions: Answer all problems in the space provided. Show all work.

1. Let A, B, C be sets. Prove $(A - B) \cup (A - C) = A - (B \cap C)$. (4 points)

Pf: Assume $x \in (A-B) \cup (A-C)$. Then $x \in A$ but $x \notin B$ or $x \in A$ but $x \notin C$. $\Rightarrow x \in A$ but $x \notin B$ or $x \notin C \Rightarrow x \notin (B \cap C) \Rightarrow x \in A - (B \cap C) \Rightarrow (A-B) \cup (A-C) \subseteq A - (B \cap C)$.
Now assume $x \in A - (B \cap C)$. Then $x \in A$ but $x \notin (B \cap C) \Rightarrow x \in A$ and $x \notin B$ or $x \notin C$. $\Rightarrow x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$, or both. Then $x \in (A-B) \cup (A-C)$, so that $A - (B \cap C) \subseteq (A-B) \cup (A-C)$. ∎

2. For sets A and B, prove that $A \times B = \emptyset$ implies $A = \emptyset$ or $B = \emptyset$. (4 points)

Pf: By the contrapositive, if $A \neq \emptyset$ and $B \neq \emptyset$, then $\exists x \in A$ and $\exists y \in B$ $\Rightarrow (x, y) \in A \times B \Rightarrow A \times B \neq \emptyset$. ∎

3. Let a be an irrational number, r a nonzero rational number. Prove that if $s \in \mathbb{R}$, then either $ar + s$ or $ar - s$ is irrational. (5 points)

Pf: Assume, for the sake of contradiction, that $ar+s$ and $ar-s$ are rational. Then $ar + s = \frac{x}{y}$ and $ar - s = \frac{m}{n}$, $x, y, m, n \in \mathbb{Z}$, $n, y \neq 0$.
By adding these equations, we get $2ar = \frac{xn+ym}{ny} \Rightarrow a = \frac{xn+ym}{2rny}$.
Since $\frac{xn+ym}{2rny} \in \mathbb{Q}$, a is rational. $\lightning$ ∎

4. Disprove: There exists an integer n such that $n^4 + n^3 + n^2 + n$ is odd. (4 points)

Pf: Note that $n^4 + n^3 + n^2 + n = n(n^3 + n^2 + n + 1)$ and we have two cases, (i) n is even. (ii) n is odd.
(i) If n is even, then $n(n^3+n^2+n+1)$ is even.
(ii) If n is odd, then n^3+n^2+n+1 is even $\Rightarrow n(n^3+n^2+n+1)$ is even.
In either case, $n^4+n^3+n^2+n$ is even. ∎

Bonus problems:

1. State the Well-Ordering Principle: The set $\mathbb{N}$ is well-ordered.