Math 308 Quiz 6

June 15, 2015

Name: ANSWERS

Instructions: Answer <u>all</u> problems in the space provided. Show all work.

1. Let $a, b \in \mathbb{Z}, a \neq 0$. Prove that $a|b \Rightarrow a^2|b^2$. (4 points)

Pf: Assume $a|b$. Then $b = ac$ for some $c \in \mathbb{Z}$.

$\Rightarrow b^2 = a^2c^2 \Rightarrow a^2|b^2$. ▨

2. Let $a, b \in \mathbb{Z}, a \neq 0, b \neq 0$. Prove that if $a|b$ and $b|a$, then $a = b$ or $a = -b$. (4 points)

Pf: Assume $a|b$ and $b|a$. Then $b = ax$ and $a = by$ for some $x, y \in \mathbb{Z}$.

Plugging the first equation into the second we get that

$a = axy \Rightarrow xy = 1 \Rightarrow$ (i) $x = 1$ and $y = 1$ or (ii) $x = -1$ and $y = -1$.

In case (i) we have $a = b$, in case (ii) we have $a = -b$. ▨

3. Let $m, n \in \mathbb{N}$ with $m \geq 2$ and $m|n$; $a, b \in \mathbb{Z}$. Prove that if $a \equiv b \ (mod\ n)$, then $a \equiv b \ (mod\ m)$. (5 points)

Pf: Assume $m|n$ and $a \equiv b \pmod{n}$. Then $n = mx$ for some $x \in \mathbb{Z}$ and

$a = b + ny$ for some $y \in \mathbb{Z}$. Then $a = b + (mx)y = b + m(xy)$.

Since $xy \in \mathbb{Z}$, we have $a \equiv b \pmod{m}$. ▨

Bonus problems:

1. Use the triangle inequality to prove that $|x| - |y| \leq |x - y|$ for $x, y \in \mathbb{R}$.

Pf: $|x| = |x - y + y| = |(x-y) + y| \underset{\uparrow\ \Delta\text{-ineq.}}{\leq} |x-y| + |y|$. ▨

2. A direct proof of $P \Rightarrow Q$ involves: 1) Assume P is true, 2) Show Q is true as a consequence. What does a proof by contradiction of $P \Rightarrow Q$ involve?

1) Assume $P \wedge (\sim Q)$ 2) Show $P \wedge (\sim Q) \Rightarrow$ Contradiction.