Math 308 Quiz 5

June 11, 2015

Name: ANSWERS

Instructions: Answer <u>all</u> problems in the space provided.

1. Prove that $5x-11$ is even if and only if x is odd. (4 points)

Pf: ($\Rightarrow$) Assume x is even. Then $x=2k$ for $k\in\mathbb{Z}$. Then $5x-11=5(2k)-11=2(5k-6)+1$. Since $(5k-6)\in\mathbb{Z}$, $5x-11$ is odd, and the ($\Rightarrow$) holds by the contrapositive.

($\Leftarrow$) Assume x is odd. Then $x=2k+1$ for $k\in\mathbb{Z}$. Then $5x-11=5(2k+1)-11=2(5k-3)$. Since $(5k-3)\in\mathbb{Z}$, $5x-11$ is even. ∎

2. Prove that if $a+b$ and ab are of the same parity, then a and b are even. (4 points)

Pf: By the contrapositive, assume a or b are odd. So (i) a and b are odd, (ii) a is odd, b is even, (iii) a is even, b is odd.

(i): If $a=2k+1$ and $b=2l+1$, then $a+b$ is even while ab is odd.

(ii) and (iii): WLOG, assume a is odd, b is even. Then $a+b$ is odd while ab is even. ∎

(We proved in class: odd + odd = even, odd + even = odd, odd × even = even, odd × odd = odd).

3. Prove that $3x+1$ is even if and only if $5x-2$ is odd. (5 points)

LEMMA: If $3x+1$ is even, then x is odd.

Pf of LEMMA: Assume, for the contrapositive, x is even. Then $x=2k$ for $k\in\mathbb{Z}$ and $3x+1=3(2k)+1=2(3k)+1$, which is odd since $3k\in\mathbb{Z}$. □

Pf of claim: ($\Rightarrow$) Assume $3x+1$ is even, then x is odd, by the lemma. This means $x=2k+1$ for $k\in\mathbb{Z}$, and so $5x-2=5(2k+1)-2=2(5k+1)+1$, which is odd since $(5k+1)\in\mathbb{Z}$.

($\Leftarrow$) Assume $5x-2$ is odd. Then x is odd, by lemma 2 (next page!). Then $x=2k+1$ for $k\in\mathbb{Z}$ and so $3x+1=3(2k+1)+1=2(3k+2)$ which is even since $(3k+2)\in\mathbb{Z}$. ∎

Bonus problems:

1. Define what it the notation $a\equiv b\ (mod\ n)$ means: $n\mid(a-b) \Leftrightarrow a=b+nk$ for $k\in\mathbb{Z}$.

2. Define what $a|b$ means: $b=ac$ for $c\in\mathbb{Z}$.

3. Define $|x|$:

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$$ assuming $x\in\mathbb{R}$.

4. Prove that $x\leq|x|$ for all $x\in\mathbb{R}$.

Pf: We have two cases: (i) $x\geq 0$ or (ii) $x<0$.

(i) If $x\geq 0$, then $|x|=x$ and so $x\leq|x|$ holds.

(ii) If $x<0$, then $|x|=-x>0$, so $x<0<|x|$, and so $x\leq|x|$ holds. ∎

Problem 3 continued!

Ahh!!! Why did I ask for "if and only if" here??

Anyway, the most straight forward way to continue is to use another lemma.

LEMMA 2: If $5x-2$ is odd, then x is odd.

Pf: Assume, by way of the contrapositive, x is even.

Then $x=2k$ for $k \in \mathbb{Z}$ and so

$5x-2 = 5(2k)-2 = 2(5k-1)$

which is even since $(5k-1) \in \mathbb{Z}$. $\square$