SAMPLE 212 FINAL EXAM

1. (a) Evaluate: $\int 5^x \tan(5^x)\,dx$

(b) Find $\dfrac{d}{dx}\left[\sqrt{\log_3 x}\right]$

(c) Does the graph of $x + x^3 + y^2 + y^4 + z^4 + z^5 = 1$ have symmetry about

(i) the x-axis? (ii) the xz-plane? (iii) the origin?

(d) What is the value of $\displaystyle\int_{-\sqrt{\pi}}^{\sqrt{\pi}} \sin(x^3)\,dx$. Explain why.

2. (a) Let the function $y(x)$ be the solution to the differential equation $y' = \dfrac{12x^3 + 12x^2}{y^2 e^{y^3}}$ for which $y(1) = 0$. Find $y(x)$ explicitly as a function of x.

(b) Find $\displaystyle\int 4x[\sec(x^2 + 2)]\,dx$.

3. (a) A package initially with a temperature of 150° F is placed in a room maintained at 70° F. Assume the temperature difference between package and the room t hours after the package was initially placed, $(\Delta T)(t) = T(t) - 70$, where $T(t)$ is the temperature of the package t hours after being placed in the room, satisfies an exponential decay law. The temperature of the package 2 hours after being placed in the room is 86° F.

(i) Find the function $(\Delta T)(t)$.

(ii) Find the temperature, expressed as a rational number (i.e., a quotient of integers), of the package after 4 hours.

(iii) How long does it take for the package to reach 71°?

(b) Evaluate: $\displaystyle\int \sec(3x + 4)\,dx$

4. (a) Evaluate: $\int_0^1 \arctan x \, dx$

(b) Evaluate: $\int_1^2 \frac{x^2 - 2x - 4}{x^3 + 2x} \, dx$

5. (a) Evaluate: $\int (\tan x + \sin^2 x \cos x) \sin 2x \, dx$

[Suggestion: Use the double angle formula for $\sin 2x$.]

(b) Evaluate: $\int \sqrt{1 - 4x^2} \, dx$

6. (a) Which of the following improper integrals is/are convergent? Show why.

(i) $\int_0^1 \frac{\ln(x+2)}{x\sqrt{x}} \, dx$ (ii) $\int_1^\infty xe^{-2x} \, dx$

(iii) $\int_1^\infty \frac{x \arctan x}{\sqrt[3]{x^7 + 7}} \, dx$

(b,3) Find $\lim_{x \to \infty} e^{\frac{x + \ln x}{\sqrt{x^2 + x + 1}}}$.

7. State, for each series, whether it converges absolutely, converges conditionally or diverges. Name a test which supports each conclusion and show the work to apply the test.

(a) $\sum_{n=1}^{\infty} (-1)^n (1 + \frac{1}{n} + \frac{1}{n^2})^n$ (b) $\sum_{n=0}^{\infty} \frac{n^2 3^n \ln(n+2)}{2^{2n+2}}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n (n^2 + 3)}{n^3 + 4}$

8. (a) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x+1)^n}{5^n\sqrt{n^2+4}}$.

Remember to check the endpoints if applicable.

(b) Sketch the graph of the polar equation $r = 3 + 2\sin\theta$, and find the area which is both inside the graph and above the x-axis.

9. (a) Find the first four nonzero terms of the Maclaurin series (i.e., power series centered at 0) for the function

$$g(x) = e^{-x^2}.$$

(b) Find the sum of the first four terms of the Maclaurin series for the derivative of the function $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n+1)^3}$.

(c) Use the answer in part (b) to approximate $f'(-\frac{1}{2})$ with an with an error of less than .01.

10. (a) Graph $4x^2 + 36y^2 + 9z^2 + 16x - 20 = 0$, and graph the trace of the answer to (a) in the xy-plane.

(b) Sketch the portion of the graph of $y = 1 - x^2$ which is in the first octant.