

MATH 209 QUIZ 2 - Version B

February 11, 2015

Name: ANSWERS

Instructions: Use your own scrap paper. Write your answers in the space provided. Simplify your answers!

1. After robbing a bank with the help of his T.A. Jennifer, Jhevon puts his share, \$25 million, in a Swiss bank account earning 15% interest annually, compounded continuously. Jhevon withdraws \$300,000 per year for "staying low" money (paying off local authorities, buying an inconspicuous Ferrari, etc.).

(For consistency, keep the units of money in single dollars, as opposed to millions of dollars, etc.) To describe the balance in Jhevon's account, write down:

(a) an ODE:  $\frac{dP}{dt} = 0.15P - 300000$  (b) An initial condition:  $P(0) = 25000000$

Solve the ODE by,

$$\frac{dP}{P - 2000000} = 0.15 dt$$

(c) First, separate the variables:  $P - 2000000$

(d) Now, solve for the general solution  $P = C e^{0.15t} + 2000000$

(e) Then, plug in the initial condition to find the particular solution.  $P = 23000000 e^{0.15t} + 2000000$

(f) How long can Jhevon "stay low"? Forever! (P will never be zero!)

(g) Jennifer deposits her money in a Dominican account earning 10% interest annually, compounded continuously. However, she continuously pulls jobs behind Jhevon's back and makes extra cash. So extra, that she can manage to deposit \$500,000 per year into her account. If her share from the robbery was \$5 million (she was only the getaway driver), write down an ODE and initial condition to describe the balance in her account.

$$\frac{dP}{dt} = 0.1P + 500000, P(0) = 5000000$$

2. We want to approximate  $\sqrt{3.9}$ .

(a) What is an appropriate function to use?  $f(x) = \sqrt{x}$

(b) Write down a general formula to make the approximation (Theorem 3.1)  $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

(c) What is your approximation to three decimal places? 1.975

3. State the equation in theorem 3.2:  $f(x+\Delta x) \approx f(x) + f'(x + \frac{\Delta x}{2}) \Delta x$

4. Consider the ODE  $\frac{dy}{dx} = x^3 + 3 + e^y$ ,  $y(0) = 0$ . Theorem 3.1 was used to obtain  $y(0.1) \approx 0.4$ . Use this fact, and theorem 3.2 to approximate  $y(0.2)$ .

$y(0.2) \approx$  0.899 (use three decimal places for the answer).

5. Consider the same ODE and initial condition from problem 4:  $\frac{dy}{dx} = x^3 + 3 + e^y$ ,  $y(0) = 0$ . Use Euler's method with step size 2 to approximate  $y(0.2)$ . Fill out the table below with the appropriate numbers, rounding to three decimal places each step where numbers exceed 3 d.p.

$x$	$y$	$y'$
0	0	4
0.1	0.4	4.493
0.2	0.849	XXXX

$\Rightarrow y(0.2) \approx$  0.849

6. Repeat problem 5, but this time, use the *modified* Euler's method.

$x$	$y$	$y'$	$x_{1/2}$	$y_{1/2}$	$y'_{1/2}$
0	0	4	0.05	0.2	4.222
0.1	0.422	4.526	0.15	0.648	4.915
0.2	0.914	XXXXX	XXXXX	XXXXX	XXXXX

$\Rightarrow y(0.2) \approx$  0.914

**Bonus:** Below, draw the *stability graph* for  $\frac{dy}{dx} = y(y - 2)$ , complete with stability arrows and smiley faces to show stability as done in class. Label the axes, steady states, and inflection points on the graph.

