MATH 209 TEST 2B
March 18, 2015

Name: JHEVON SMITH

Note that both sides of each sheet have printed material.

Instructions:

1. Read the instructions.
2. Don't panic!
3. Complete all problems! Bonus problems will not be counted unless all problems in the actual test are completed.
4. Note that each problem in the test is worth 20 points. The point values of the bonus problems are indicated.
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answer.
6. Write neatly, so that I am able to follow your sequence of steps. Indicate your answers by boxing them or otherwise.
7. Read through the exam and kill all the easy problems (for you) first!
8. Scientific calculators are needed, but you are NOT allowed to use notes, phones (especially iPhones!), iPads, telepathy, divine inspiration, or other outside aids--including, but not limited to, the smart kid that may be sitting beside you, or the friend you might be thinking of texting.
9. In fact, cell phones should be out of sight. Especially iPhones.
10. Use correct notation and write what you mean! "x^2" and "$x2$" are NOT the same thing, for example. I will grade accordingly.
11. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding. From test 1, you guys know I'm not kidding.
12. Other than that, have fun, and good luck! :)

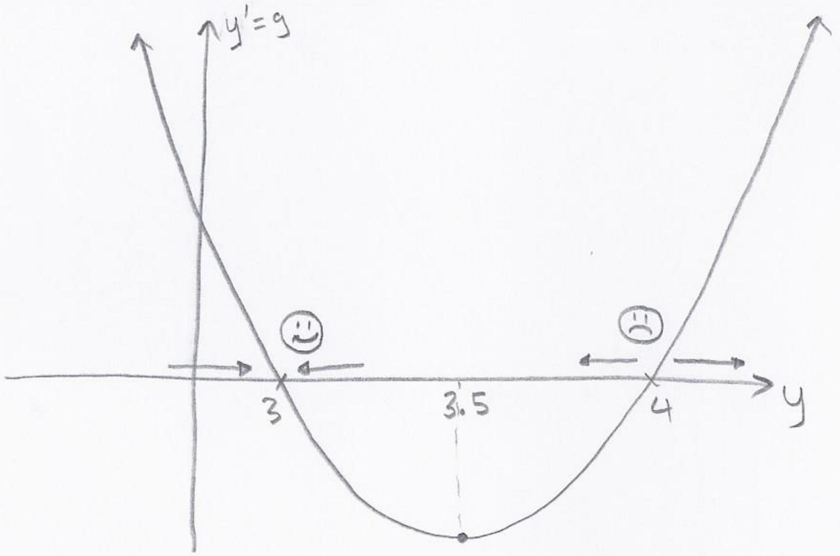
Remember: ...because this is the test this class deserves, and also the one it needs right now.

1. Use geometric analysis to analyze the differential equation $\frac{dy}{dt} = y(y-7) + 12$, by:

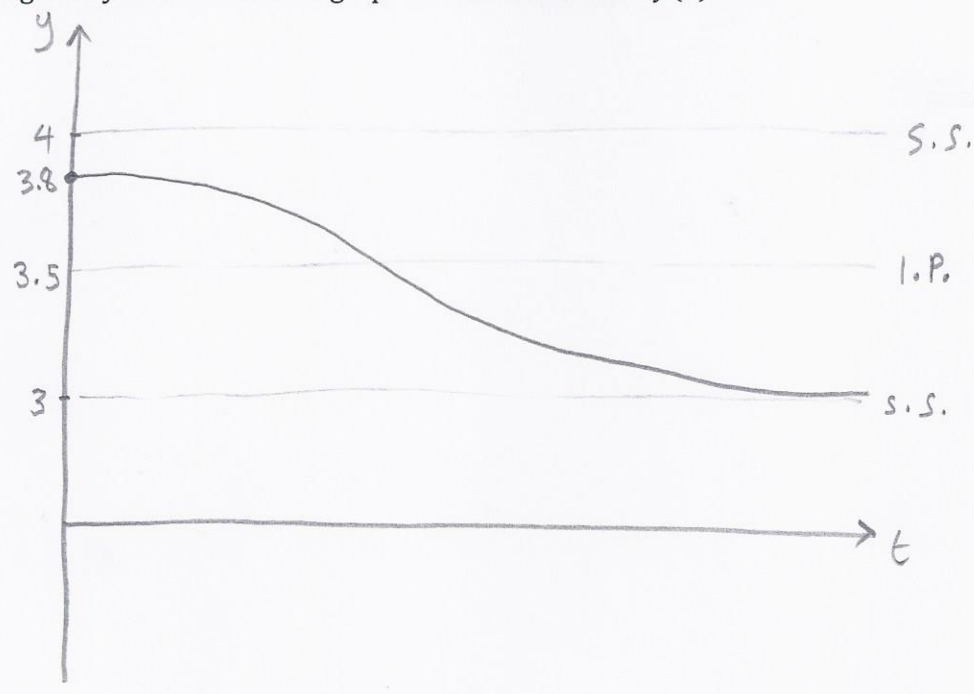
(i) Finding the steady state solutions.

$$g = y(y-7)+12$$
$$= y^2 - 7y + 12$$
$$= (y-4)(y-3)$$
$$\Rightarrow \boxed{y=4,\ y=3 \text{ are S.S.}}$$

(ii) Drawing a fully labeled stability graph, complete with stability arrows.



(iii) Sketching a fully labeled solution graph for the initial value $y(0) = 3.8$.



2. Use geometric analysis to analyze the differential equation $\frac{dy}{dt} = (y-1)(y+2)^2$, by:

(i) Finding the steady state solutions.

$g = (y-1)(y+2)^2$

$\Rightarrow \boxed{y = 1,\ y = -2 \text{ are S.S.}}$

For graph: inflection points

$g = (y-1)(y^2+4y+4)$

$= y^3 + 4y^2 + 4y - y^2 - 4y - 4$

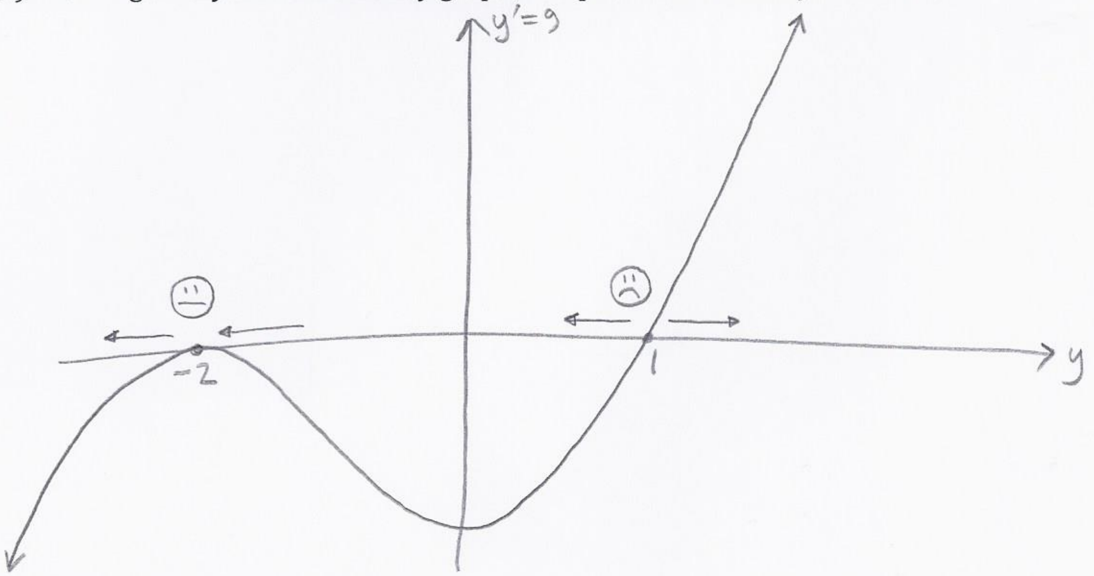
$= y^3 + 3y^2 - 4$

$\Rightarrow g' = 3y^2 + 6y$

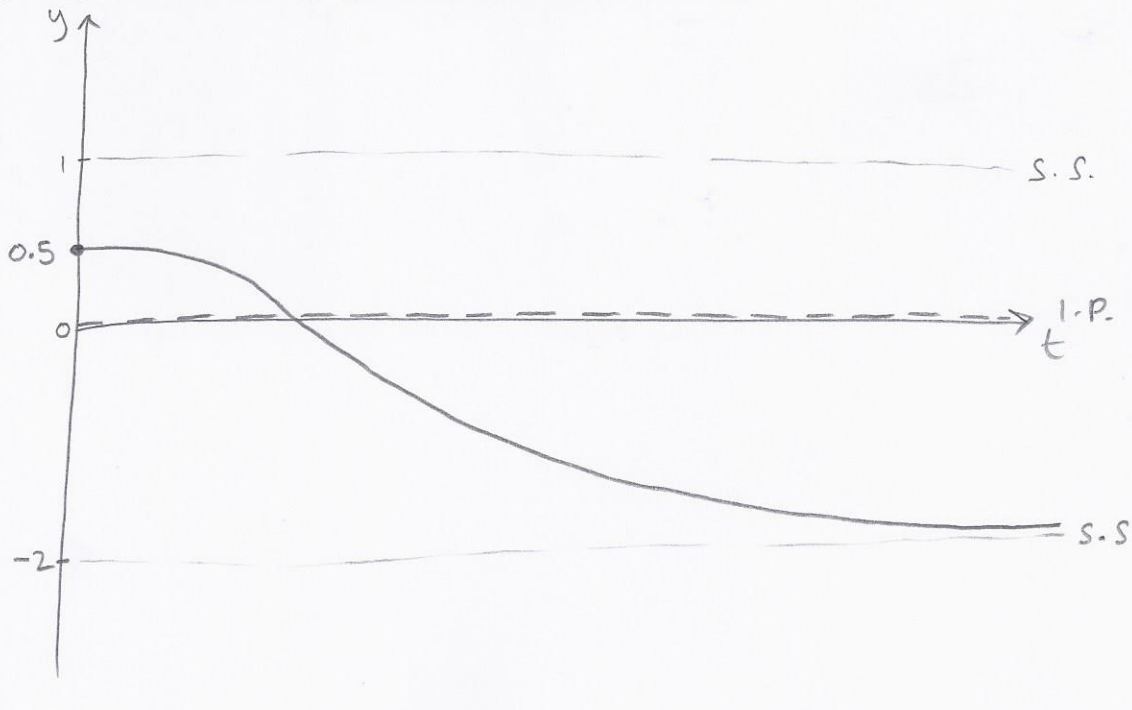
$= 3y(y+2)$

$\Rightarrow y = 0,\ y = -2$ are the I.P.

(ii) Drawing a fully labeled stability graph, complete with stability arrows.



(iii) Sketching a fully labeled solution graph for the initial value $y(0) = 0.5$.



3. An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\frac{dN_1}{dt} = 0.57N_1\left(1 - \frac{N_1}{50} - \frac{N_2}{25}\right) \quad (A)$$
$$\frac{dN_2}{dt} = 0.26N_2\left(1 - \frac{N_2}{75} - \frac{N_1}{25}\right) \quad (B)$$

(a) Find all steady-state solutions of this system.

In (A): $N_1 = 0$ or $1 - \frac{N_1}{50} - \frac{N_2}{25} = 0$

$\Rightarrow N_1 = 50 - 2N_2$

In (B): If $N_1 = 0$

$\Rightarrow N_2 = 0$ or $1 - \frac{N_2}{75} = 0$

$N_2 = 75$

$\Rightarrow \boxed{(0,0)\ ,\ (0,75) \text{ are S.S.}}$

If $N_1 = 50 - 2N_2$

$\Rightarrow N_2 = 0$ or $1 - \frac{N_2}{75} - \frac{(50-2N_2)}{25} = 0$

$\Rightarrow 75 - N_2 - 3(50 - 2N_2) = 0$

$\Rightarrow 75 - N_2 - 150 + 6N_2 = 0$

$\Rightarrow -75 + 5N_2 = 0$

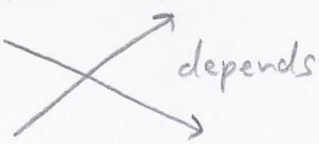
$\Rightarrow N_2 = 15$

$\Rightarrow \boxed{(50,0),\ (20,15) \text{ are S.S.}}$

(b) State and justify whether or not the species are competitive.

Yes, they are competitive, by:

Lo's method

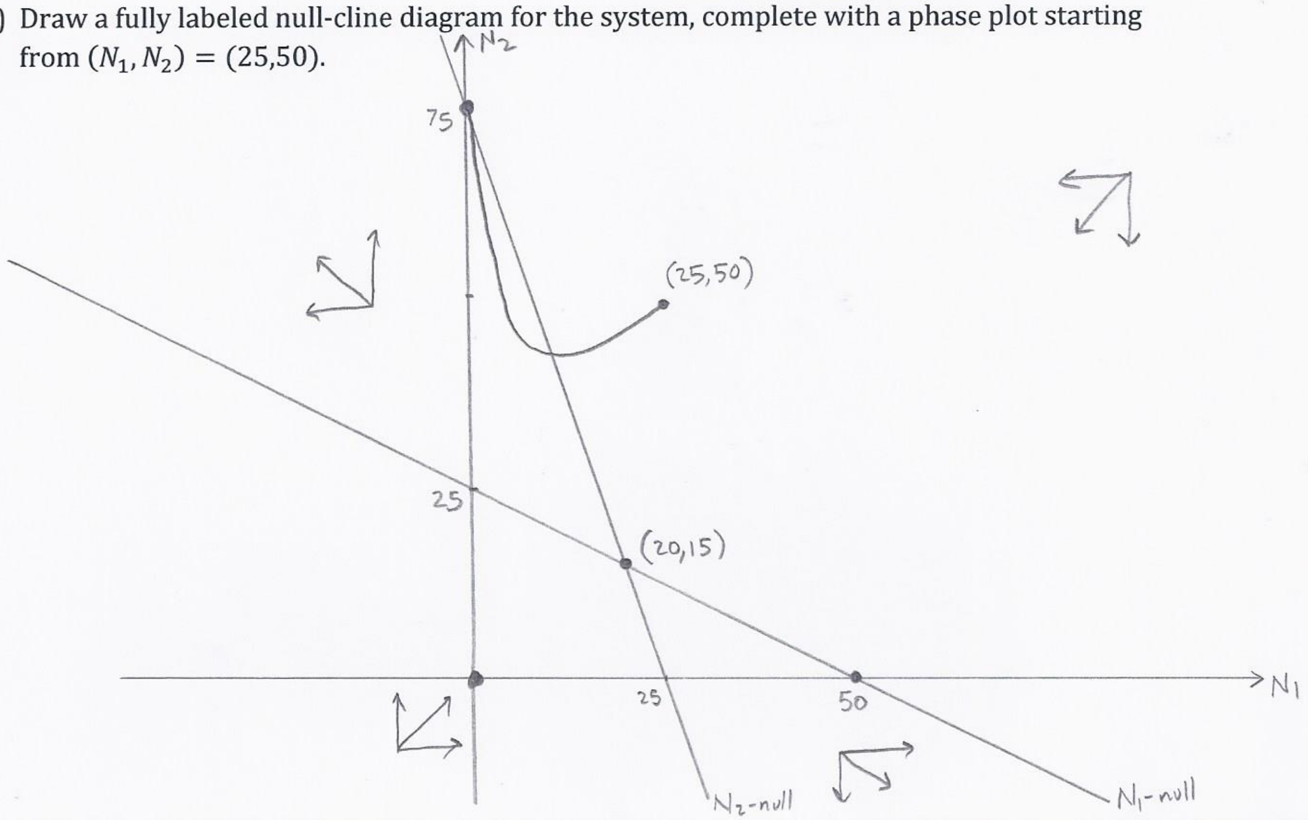


OR

Competition table

	1	2
1	$\frac{1}{50} = 0.02$	$< \frac{1}{25} = 0.04$
2	$\frac{1}{25} = 0.04$	$> \frac{1}{75} = 0.013$
Total	0.06	0.053

(c) Draw a fully labeled null-cline diagram for the system, complete with a phase plot starting from $(N_1, N_2) = (25,50)$.



4. Jennifer has a SpongeBob fish farm in her back yard. She has 6000 fish that reproduce at a rate of 6% per year. She harvests 600 fish per year to have with her platanos and salami mega lunch.

(a) Write down a differential equation, with initial condition, to model Jennifer's SpongeBob population.

$$\frac{dN}{dt} = 0.06N - 600, \qquad N(0) = 6000$$

(b) Analyze the population qualitatively by:

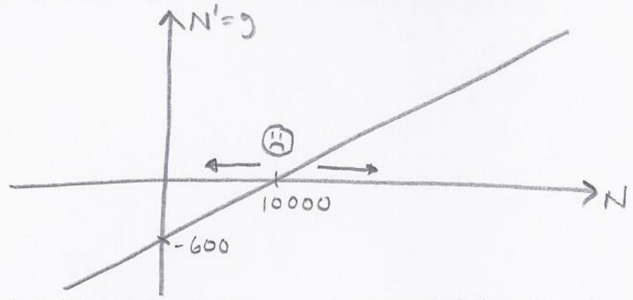
(i) Finding the steady states.

$g(N) = 0.06(N - 10000)$

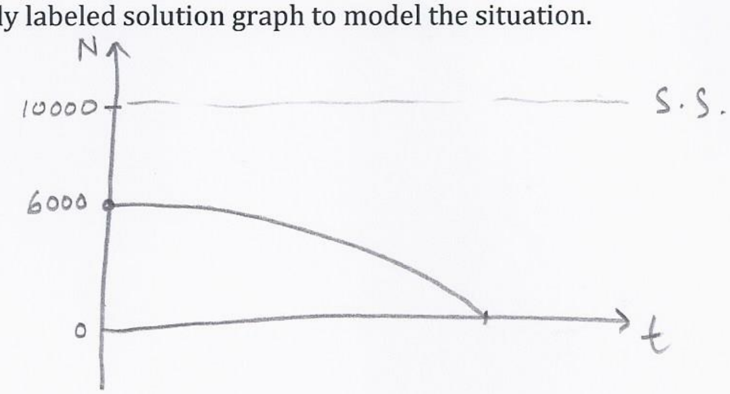
$\Rightarrow \boxed{N = 10000 \text{ is S.S.}}$

(Note: $g' = 0.06 \neq 0$ So no inflections).

(ii) Sketching a fully labeled stability graph with stability arrows.



(iii) Sketching a fully labeled solution graph to model the situation.



(c) Can Jennifer continue to have SpongeBob platanos and salami indefinitely?

No

(d) If not, when will she run out of SpongeBob?

$\frac{dN}{dt} = 0.06(N - 10000)$

$\Rightarrow \int \frac{dN}{N - 10000} = \int 0.06\, dt$

$\Rightarrow \ln|N - 10000| = 0.06t + C$

$\Rightarrow N - 10000 = Ce^{0.06t}$

$\Rightarrow N = Ce^{0.06t} + 10000$

$N(0) = 6000 \Rightarrow C = -4000$

$\Rightarrow N = -4000e^{0.06t} + 10000$

when $N = 0$, we get

$0 = -4000e^{0.06t} + 10000$

$\Rightarrow 2.5 = e^{0.06t}$

$\Rightarrow t = \frac{\ln(2.5)}{0.06}$

$\approx \boxed{15.27 \text{ years later}}$