Math 205 JW Quiz 9

October 31, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let $f(x)$ and $g(x)$ be differentiable functions of x, a a constant. Complete the following formulas. (You may use $f'$and g' as shorthand):

(a) $\frac{d}{dx}(x^n) =$ nx^{n-1} (b) $\frac{d}{dx}e^u =$ $u'e^u$ (c) $\frac{d}{dx}(a^u) =$ $u'a^u \ln a$

(d) $\frac{d}{dx}\ln u =$ $\frac{u'}{u}$ (e) $\frac{d}{dx}(f(x)\cdot g(x)) =$ $f'g + fg'$

(f) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$ $\frac{f'g - fg'}{g^2}$ (g) $\frac{d}{dx}f(g(x)) =$ $f'(g(x))\cdot g'(x)$

2. Differentiate, or find y' in, the following (you don't have to simplify):

(a) $\frac{d}{dx}\frac{\sqrt{x}(x+1)}{x^2e^x} =$ $y\left(\frac{1}{2x} + \frac{1}{x+1} - \frac{2}{x} - 1\right)$ * (b) $\frac{d}{dx}xe^x\ln x =$ $e^x\ln x + xe^x\ln x + e^x$

(c) $\frac{d}{dx}\left[x^x + e^{x^3}\right] =$ $x^x(\ln x+1) + 3x^2e^{x^3}$ (d) $xy + \frac{x}{y} + e^y + \ln x = x^2 \Rightarrow y' =$ $\frac{2x - y - \frac{1}{y} - \frac{1}{x}}{x - \frac{x}{y^2} + e^y}$

3. If $c(x) = 3 + \frac{2}{x}$ is a cost function, what is the marginal cost function? $C'(x) = -2x^{-2}$

4. Use linear approximation to estimate $\sqrt{8.9}$ by doing the following:

(a) Write down the linear approximation formula: $f(x) \approx f(a) + f'(a)(x-a)$

(b) Define an appropriate function: $f(x) = \sqrt{x}$

(c) What are the values for $x =$ 8.9 and $a =$ 9

(d) Compute; and write your answer as a fraction: $\frac{179}{60}$

Bonus:

1. The half-life of an ingredient in Jhevon's hotdogs is 128 days. If the ingredient decays radioactively, answer the following; assuming you have 2 pounds of the ingredient and $P(t)$ represents the amount of the ingredient at time t.

(a) Write down the differential equation with initial condition for the "secret" ingredient:

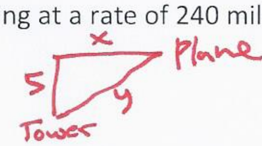
Differential equation: $P' = -\frac{\ln 2}{128}P$ Initial condition: $P(0) = 2$

(b) A formula for $P(t)$ is: $P(t) =$ $2e^{-\frac{\ln 2}{128}t}$

(c) After how long will there be 0.2 pounds of the ingredient? You may leave e's and $\ln$'s in your answer: $t =$ $\frac{-128\ln 0.1}{\ln 2}$

2. An airplane flying at an altitude of 5 miles passes directly over a radar tower. When the distance between the tower and the airplane is 10 miles, the tower detects that its distance from the plane is changing at a rate of 240 miles per hour. How fast is the plane flying?

The equation I used (before differentiating) is $y^2 = x^2 + 5^2$ (or similar)



After differentiating I have $2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

Therefore, the plane is traveling at a speed of $480/\sqrt{3}$ miles per hour.

* $y = \frac{\sqrt{x}(x+1)}{x^2e^x}$