Math 205 Test 4
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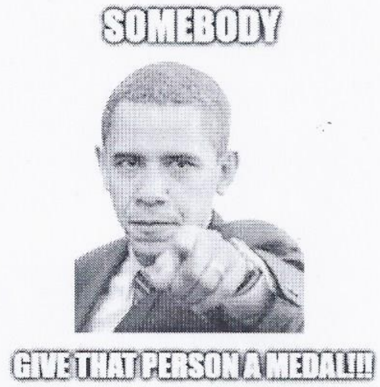
Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are needed, but you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Other than that, have fun and good luck!

You survived to the end of Math 205??



1. (10 points each) Evaluate the following integrals. Simplify your answers.

(a) $\int \frac{2}{3x\sqrt{\ln x}}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\Rightarrow dx = x\,du$

$\Rightarrow \frac{2}{3}\int \frac{1}{x\sqrt{u}}\,x\,du$

$= \frac{2}{3}\int u^{-1/2}\,du$

$= \frac{2}{3}\cdot 2u^{1/2} + C$

$= \boxed{\frac{4}{3}(\ln x)^{1/2} + C}$

(b) $\int 5x^{-3}\left(\frac{1}{x^2} - 7\right)^4 dx$

$u = \frac{1}{x^2} - 7$

$du = -2x^{-3}\,dx$

$\Rightarrow dx = -\frac{1}{2}x^3\,du$

$\Rightarrow \int 5\cancel{x^{-3}}\cdot u^4 \cdot -\frac{1}{2}\cancel{x^3}\,du$

$= -\frac{5}{2}\int u^4\,du$

$= -\frac{1}{2}u^5 + C$

$= \boxed{-\frac{1}{2}\left(\frac{1}{x^2} - 7\right)^5 + C}$

(c) $\int \frac{e^{4x} - x}{3} dx$

$= \frac{1}{3}\int e^{4x} - x \, dx$

$= \frac{1}{3}\left[\frac{1}{4}e^{4x} - \frac{x^2}{2}\right] + C$

$= \boxed{\frac{1}{12}e^{4x} - \frac{x^2}{6} + C}$

(d) $\int_0^{\ln 3} \frac{2e^x}{(1+e^x)^3} dx$

$u = 1 + e^x$

$du = e^x dx$

$\Rightarrow dx = \frac{1}{e^x} du$

$\Rightarrow \int_{x=0}^{\ln 3} \frac{2\cancel{e^x}}{u^3} \cdot \frac{1}{\cancel{e^x}} du$

$= 2\int_{x=0}^{\ln 3} u^{-3} du$

$= -u^{-2}\Big|_{x=0}^{\ln 3}$

$= -(1+e^x)^{-2}\Big|_0^{\ln 3}$

$= -(1+e^{\ln 3})^{-2} + (1+e^0)^{-2}$

$= \boxed{-4^{-2} + 2^{-2}}$ or $-\frac{1}{16} + \frac{1}{4} = \boxed{\frac{3}{16}}$

(e) $\int \frac{x^2 - x}{2x^3 - 3x^2 + 1}\, dx$

$u = 2x^3 - 3x^2 + 1$

$du = (6x^2 - 6x)\, dx$

$du = 6(x^2 - x)\, dx$

$\Rightarrow dx = \frac{du}{6(x^2 - x)}$

$\Rightarrow \int \frac{\cancel{x^2 - x}}{u} \cdot \frac{du}{6\cancel{(x^2 - x)}}$

$= \frac{1}{6} \int \frac{1}{u}\, du$

$= \frac{1}{6} \ln|u| + C$

$= \boxed{\frac{1}{6} \ln|2x^3 - 3x^2 + 1| + C}$

2. (30 points) Find the area of the region bounded by the curves $y_1 = x^2 + 8x - 1$ and $y_2 = 2x - 6$. You need not sketch the region—but it might come in handy.

Intersections

$x^2 + 8x - 1 = 2x - 6$

$\Rightarrow x^2 + 6x + 5 = 0$

$\Rightarrow (x+5)(x+1) = 0$

$\Rightarrow x = -1,\ x = -5$

$\Longrightarrow A = \int_{-5}^{-1} 2x - 6 - (x^2 + 8x - 1)\, dx$

$= \int_{-5}^{-1} -x^2 - 6x - 5\, dx$

$= -\frac{x^3}{3} - 3x^2 - 5x \Big|_{-5}^{-1}$

$= \frac{1}{3} - 3 + 5 + \frac{125}{3} + 75 - 25$

$= \boxed{\frac{32}{3}}$

$y_1 = (-2)^2 + 8(-2) - 1$

$= 4 - 16 - 1$

$= -13$

$y_2 = 2(-2) - 6$

$= -4 - 6$

$= -10 \rightarrow$ top!

3. (20 points) Use Riemann sums to approximate the net area under the curve $y = 25 - x^2$ on the interval $0 \leq x \leq 4$ using $n = 4$ subintervals and *right hand endpoints*.

$$\Delta x = \frac{4-0}{4} = 1$$

$$A \approx R_4 = \Delta x\left(f(1)+f(2)+f(3)+f(4)\right)$$

$$= 1\left[25-1^2+25-2^2+25-3^2+25-4^2\right]$$

$$= 100-1-4-9-16$$

$$= \boxed{70}$$

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. (10 points) The velocity of a moving particle is given by $v(t) = 6t^2 - 4$, where t is time in seconds, and velocity is measured in meters per second. Find its position function, $s(t)$, if you know that $s(2) = 10$.

$$s(t) = \int v(t)\,dt$$
$$= \int 6t^2 - 4\,dt$$
$$= 2t^3 - 4t + C$$

since $s(2) = 10$

$$\Rightarrow 10 = 2(2)^3 - 4(2) + C$$
$$10 = 8 + C$$
$$\Rightarrow C = 2$$
$$\Rightarrow \boxed{s(t) = 2t^3 - 4t + 2}$$

2. (10 points) A particle moves along the circle $x^2 + y^2 = 8$. As the particle passes through the point (2,2), its x-coordinate is *decreasing* at a rate of 2 units per second. At what rate is the y-coordinate changing at this instant? Is it increasing or decreasing?

$$x^2 + y^2 = 8$$
$$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$\Rightarrow \cancel{2}(\cancel{2})(-2) + \cancel{2}(\cancel{2})\frac{dy}{dt} = 0$$
$$\Rightarrow -2 + \frac{dy}{dt} = 0$$
$$\Rightarrow \boxed{\frac{dy}{dt} = 2}$$