Math 205 Test 2
July 5, 2016

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Note that both sides of each page may have printed material.

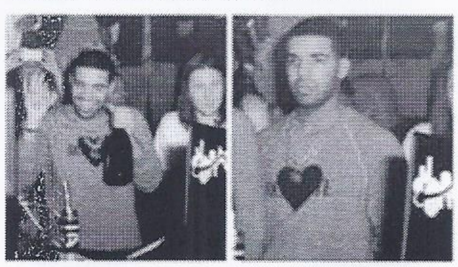
Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are needed, but you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Other than that, have fun and good luck!

Uncle Sam wants YOU to get a good grade on this test.

When ur having fun on July 4th.

And you remember you have a Math 205 test tomorrow.



1. (a) (15 points) Let $f(x) = 2x - \frac{3}{x}$. Use the limit definition of the derivative to find $f'(x)$. **No credit will be given for any other method!**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h) - \frac{3}{x+h} - \left(2x - \frac{3}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x} + 2h - \frac{3}{x+h} - \cancel{2x} + \frac{3}{x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{2\cancel{h}}{\cancel{h}} + \frac{\frac{3}{x} - \frac{3}{x+h}}{h} \right)$$

$$= \lim_{h\to 0} \left(2 + \frac{\frac{3}{x} - \frac{3}{x+h}}{h} \cdot \frac{x(x+h)}{x(x+h)} \right)$$

$$= \lim_{h\to 0} \left(2 + \frac{3(x+h) - 3x}{h \cdot x(x+h)} \right)$$

$$= \lim_{h\to 0} \left(2 + \frac{\cancel{3x} + 3\cancel{h} - \cancel{3x}}{\cancel{h}x(x+h)} \right)$$

$$= \lim_{h\to 0} \left(2 + \frac{3}{x(x+h)} \right)$$

$$= \boxed{2 + \frac{3}{x^2}}$$

(b) (5 points) Using your answer to part (a), compute the equation of the tangent line to $f(x)$ at the point where $x = 1$. **Write your line in $y = mx + b$ form.**

$$m = f'(1) = 2 + \frac{3}{1^2} = 5$$

$$y = f(1) = 2(1) - \frac{3}{1} = -1$$

tangent line: $y - y_1 = m(x - x_1)$

$$\Rightarrow y + 1 = 5(x - 1)$$

$$\Rightarrow \boxed{y = 5x - 6}$$

2. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (4 points each)

(a) $y = \frac{5x^3+3x^2}{2x}$ $= \frac{5x^3}{2x} + \frac{3x^2}{2x}$

$= \frac{5}{2}x^2 + \frac{3}{2}x$ ⟶ Power Rule!

$\Rightarrow \boxed{y' = 5x + \frac{3}{2}}$

(b) $y = 2\sqrt{x} + \frac{5}{\sqrt[3]{x}} - \ln(x^2+1)^3 - \pi^2$

$= 2x^{\frac{1}{2}} + 5x^{-1/3} - 3\ln(x^2+1) - \pi^2$ ⟶ Power Rule and log. diff. rule!

$\Rightarrow y' = x^{-1/2} - \frac{5}{3}x^{-4/3} - 3\cdot\frac{2x}{x^2+1}$

$= \boxed{x^{-1/2} - \frac{5}{3}x^{-4/3} - \frac{6x}{x^2+1}}$

(c) $y = \frac{x^5}{5+x^5}$ ⟶ Quotient Rule!

$\Rightarrow y' = \frac{5x^4(5+x^5) - x^5(5x^4)}{(5+x^5)^2}$

$= \frac{5x^4[5+\cancel{x^5}-\cancel{x^5}]}{(5+x^5)^2}$

$= \boxed{\frac{25x^4}{(5+x^5)^2}}$

(d) $y = x^2\sqrt{2^x - 1} + x^{x^2}$

One way:

$y = x^2(2^x-1)^{1/2} + e^{\ln x^{x^2}}$

$= x^2(2^x-1)^{1/2} + e^{x^2\ln x}$

$\Rightarrow y' = 2x(2^x-1)^{1/2} + x^2\cdot\frac{1}{2}(2^x-1)^{-1/2}(2^x\ln 2) + (2x\ln x + x)e^{x^2\ln x}$

$= \boxed{2x(2^x-1)^{1/2} + x^2(2^x-1)^{-1/2}2^{x-1}\ln 2 + (2x\ln x + x)x^{x^2}}$

(See next page for another way to do this!)

2. Find $\frac{dy}{dx} = y'$ for the following. Simplify your answers. (4 points each)

(a) $y = \frac{5x^3+3x^2}{2x}$

(b) $y = 2\sqrt{x} + \frac{5}{\sqrt[3]{x}} - \ln(x^2+1)^3 - \pi^2$

(c) $y = \frac{x^5}{5+x^5}$

(d) $y = x^2\sqrt{2^x - 1} + x^{x^2}$

Another way:

First do log diff. for

$g = x^{x^2}$

$\Rightarrow \ln g = \ln x^{x^2}$

$\Rightarrow \ln g = x^2 \ln x$

$\Rightarrow \frac{g'}{g} = 2x\ln x + x$

$\Rightarrow g' = g(2x\ln x + x)$

$= x^{x^2}(2x\ln x + x)$

$$\Rightarrow y' = 2x(2^x-1)^{1/2} + x^2\frac{1}{2}(2^x-1)^{-1/2}(2^x\ln 2)$$

$$+$$

$$x^{x^2}(2x\ln x + x)$$

$$\Rightarrow \boxed{y' = 2x(2^x-1)^{1/2} + x^2(2^x-1)^{-1/2}2^{x-1}\ln 2 + x^{x^2}(2x\ln x + x)}$$

(e) $x^2y + 2x + 3y = 7x + 12$

$\Rightarrow 2xy + x^2y' + 2 + 3y' = 7$

$\Rightarrow y'(x^2+3) = 7 - 2xy - 2$

$\Rightarrow \boxed{y' = \frac{5-2xy}{x^2+3}}$

3. (5 points each part) A bunch of angry calculus students (allegedly) throw Jhevon off a cliff. Jhevon's position above the ground at time t seconds is given by $s(t) = -16t^2 + 16t + 96$.

(a) Find functions that describe Jhevon's velocity and acceleration at time t.

Velocity function $= \boxed{v(t) = -32t + 16}$

Acceleration function $= \boxed{a(t) = -32}$

(b) What is the highest height that Jhevon attains?

One way:

Find vertex of $s(t)$:

$t = \frac{-b}{2a} = \frac{-16}{2(-16)} = \frac{1}{2}$ second.

$\Rightarrow$ max height $= s(\frac{1}{2})$

$= -16(\frac{1}{2})^2 + 16(\frac{1}{2}) + 96$

$= -4 + 8 + 96$

$= \boxed{100 \text{ ft}}$

Another way:

Max height $\Rightarrow v(t) = 0$

$\Rightarrow -32t + 16 = 0$

$\Rightarrow t = 1/2$

$\Rightarrow$ max height $= s(\frac{1}{2})$

$= \boxed{100 \text{ ft}}$

(c) When will Jhevon hit the ground and the nightmare end for his students?

We want when $s(t) = 0$

$\Rightarrow -16t^2 + 16t + 96 = 0$

$\Rightarrow t^2 - t - 6 = 0$

$\Rightarrow (t-3)(t+2) = 0$

$\Rightarrow \boxed{t = 3}$, $\cancel{t = -2}$ reject! Negative time.

$\Rightarrow$ Jhevon hits the ground after $t = 3$ seconds