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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are needed, but you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly..
10. Other than that, have fun and good luck!

May the force be with you. But you can't ask it to help you with your test.

GET 120 ON THIS TEST??



CHALLENGE ACCEPTED!!

1. (5 points each) Simplify the following:

$$(a) \frac{5(x^2+2)^4(2x)(x^3-7)^6 - (x^2+2)^5(6)(x^3-7)^5(3x^2)}{(x^3-7)^{12}}$$

$$= \frac{2x(x^2+2)^4 \cancel{(x^3-7)^5} [5(x^3-7) - (x^2+2)(3)(3x)]}{(x^3-7)^{12-5}}$$

$$= \frac{2x(x^2+2)^4 [5x^3 - 35 - 9x^3 - 18x]}{(x^3-7)^7}$$

$$= \frac{2x(x^2+2)^4 (-4x^3 - 18x - 35)}{(x^3-7)^7}$$

$$\text{OR } \frac{-2x(x^2+2)^4 (4x^3 + 18x + 35)}{(x^3-7)^7}$$

$$(b) \ln \sqrt{\frac{2e^{5x}\sqrt{x}}{x^3(x+1)^2}} = \frac{1}{2} \ln \left(\frac{2e^{5x}x^{1/2}}{x^3(x+1)^2} \right)$$

$$= \frac{1}{2} [\ln 2 + \ln e^{5x} + \frac{1}{2} \ln x - 3 \ln x - 2 \ln(x+1)]$$

$$= \frac{1}{2} (\ln 2 + 5x - \frac{5}{2} \ln x - 2 \ln(x+1))$$

$$(c) e^{2 \ln(5x) - 3 + (\frac{1}{2}) \ln y}$$

$$= e^{\ln(5x)^2} \cdot e^{-3} \cdot e^{\ln y^{1/2}}$$

$$= \frac{(5x)^2 y^{1/2}}{e^3}$$

$$(d) \frac{\frac{1}{(x+h)^3} \frac{1}{x^3}}{h} \cdot \frac{x^3(x+h)^3}{x^3(x+h)^3} = \frac{x^3 - (x+h)^3}{h x^3(x+h)^3}$$

$$= \frac{x^3 - x^3 - 3x^2h - 3xh^2 - h^3}{h x^3(x+h)^3}$$

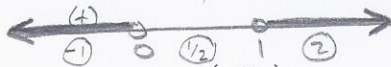
$$= \frac{-h(3x^2 + 3xh + h^2)}{h x^3(x+h)^3}$$

$$= \frac{-(3x^2 + 3xh + h^2)}{x^3(x+h)^3}$$

2. (a) (4 points each) Find the domains of the following functions:

(i) $f(x) = \frac{3x+4}{\sqrt{x^2-x}}$ We need $x^2-x \geq 0$ and $x^2-x \neq 0 \Rightarrow x^2-x > 0$

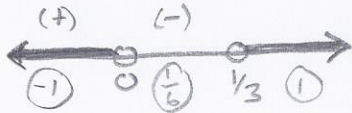
If $x^2-x=0$
 $\Rightarrow x(x-1)=0$
 $\Rightarrow x=0, x=1$



$\Rightarrow \text{dom}(f) = (-\infty, 0) \cup (1, \infty)$

(ii) $g(x) = \ln\left(\frac{x}{3x-1}\right)$

We need $\frac{x}{3x-1} > 0$. Set $x=0, 3x-1=0 \Rightarrow x=1/3$:



$\Rightarrow \text{dom}(g) = (-\infty, 0) \cup (1/3, \infty)$

(b) (4 points) Find the equation of the line that passes through $(1, -2)$ that is perpendicular to $3x - 2y = 4 \Rightarrow y = \frac{3}{2}x - 2 \Rightarrow m_{\text{new}} = -2/3$

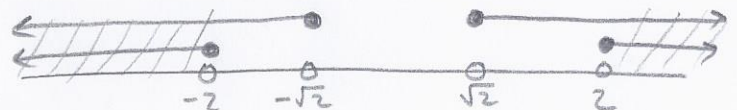
$\Rightarrow y + 2 = -\frac{2}{3}(x - 1)$ or $y = -\frac{2}{3}x - \frac{4}{3}$

(c) (4 points each) Find and simplify the indicated compositions, given $f(x) = \sqrt{2x^2 + 4}$ and $g(x) = \sqrt{x^2 - 4}$, and state the domains of each composite function:

(i) $f \circ g = f(g(x))$
 $= \sqrt{2(\sqrt{x^2-4})^2 + 4}$
 $= \sqrt{2x^2 - 8 + 4}$
 $= \sqrt{2(x^2 - 2)}$

Note: $\text{dom}(g) = (-\infty, -2] \cup [2, \infty)$

$\text{dom}(\sqrt{2(x^2-2)}) = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$



Domain of $f \circ g$: $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

(ii) $g \circ f = g(f(x))$
 $= \sqrt{(\sqrt{2x^2+4})^2 - 4}$
 $= \sqrt{2x^2 + 4 - 4}$
 $= \sqrt{2x^2}$

Note: $\text{dom}(f) = (-\infty, \infty)$

$\text{dom}(\sqrt{2x^2}) = (-\infty, \infty)$

Domain of $g \circ f$: $(-\infty, \infty)$

3. (a) (4 points) Let $f(x)$ be a function, state its difference quotient.

$$\frac{f(x+h) - f(x)}{h}$$

or

$$\frac{f(b) - f(a)}{b - a}$$

(b) (8 points each) Find and simplify the difference quotient of the following functions:

(i) $f(x) = 2 - x^2$

Using $\frac{f(x+h) - f(x)}{h}$

$$= \frac{2 - (x+h)^2 - (2 - x^2)}{h}$$

$$= \frac{2 - x^2 - 2xh - h^2 - 2 + x^2}{h}$$

$$= \frac{-2xh - h^2}{h}$$

$$= \boxed{-2x - h}$$

OR

Using $\frac{f(b) - f(a)}{b - a}$

$$= \frac{2 - b^2 - (2 - a^2)}{b - a}$$

$$= \frac{2 - b^2 - 2 + a^2}{b - a}$$

$$= \frac{a^2 - b^2}{b - a}$$

$$= \frac{(a-b)(a+b)}{b - a}$$

$$= \boxed{-(a+b)}$$

(ii) $f(x) = 1 - \frac{2}{x}$

Using $\frac{f(x+h) - f(x)}{h}$

$$= \frac{1 - \frac{2}{x+h} - (1 - \frac{2}{x})}{h}$$

$$= \frac{-\frac{2}{x+h} + \frac{2}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{-2x + 2(x+h)}{hx(x+h)}$$

$$= \frac{-2x + 2x + 2h}{hx(x+h)}$$

$$= \boxed{\frac{2}{x(x+h)}}$$

OR

Using $\frac{f(b) - f(a)}{b - a}$

$$= \frac{1 - \frac{2}{b} - (1 - \frac{2}{a})}{b - a}$$

$$= \frac{-\frac{2}{b} + \frac{2}{a}}{b - a} \cdot \frac{ab}{ab}$$

$$= \frac{-2a + 2b}{(b-a)ab}$$

$$= \frac{2(b-a)}{(b-a)ab}$$

$$= \boxed{\frac{2}{ab}}$$

