Math 205 Quiz 5

July 11, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let $f(x)$, $g(x)$ and u be differentiable functions of x, c a constant. Complete the following formulas. (You may use f', g' and u' as shorthand):

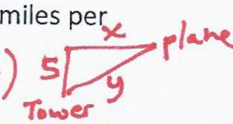
(a) $\frac{d}{dx}(cf(x)) =$ $c \cdot f'(x)$ (b) $\frac{d}{dx}(f(x)\cdot g(x)) =$ $f'g + fg'$ (c) $\frac{d}{dx}e^u =$ $u'e^u$

(d) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$ $\frac{f'g - fg'}{g^2}$ (e) $\frac{d}{dx}\ln u =$ $\frac{u'}{u}$ (f) $\frac{d}{dx}f(g(x)) =$ $f'(g(x)) \cdot g'(x)$

(g) $\frac{d}{dx}x^n =$ nx^{n-1} (h) $\frac{d}{dx}a^u =$ $u'a^u \ln a$

2. An airplane flying at an altitude of 5 miles passes directly over a radar tower. When the distance between the tower and the airplane is 10 miles, the tower detects that its distane from the plane is changing at a rate of 240 miles per hour. How fast is the plane flying?

The equation I used (before differentiating) is $y^2 = x^2 + 5^2$ (or something similar)



After differentiating I have $2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

Therefore, the plane is traveling at a speed of $480/\sqrt{3}$ miles per hour.

3. Find x-values of the critical points (if they exist) of the function $f(x) = 3x^4 - 6x^3$. List the x-values separated by commas below. If there are none, write "none".

Critical points: $x =$ $0, 3/2$

4. For the function above, find the absolute extrema on [1,2].

Absolute maximum(s): $f(2) = 0$

Absolute minimum(s): $f(\frac{3}{2}) = -\frac{81}{16}$

5. The half-life of an ingredient in Jhevon's hotdogs is 128 days. If the ingredient decays radioactively, answer the following; assuming you have 2 pounds of the ingredient and $P(t)$ represents the amount of the ingredient at time t.

(a) Write down the differential equation with initial condition for the "secret" ingredient:

Differential equation: $P' = -\frac{\ln 2}{128}P$ Initial condition: $P(0) = 2$

(b) A formula for $P(t)$ is: $P(t) =$ $P = 2e^{-\frac{\ln 2}{128}t}$

(c) After how long will there be 0.2 pounds of the ingredient? You may leave e's and $\ln$'s in your answer: $t =$ $\frac{-128 \ln 0.1}{\ln 2}$

Bonus:

1. (3 points) Provide a naïve sketch of the graph from problem 3. You need not consider concavity. Plot intercepts and extrema.