

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

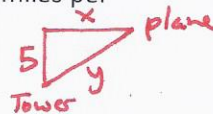
1. Let  $f(x)$ ,  $g(x)$  and  $u$  be differentiable functions of  $x$ ,  $c$  a constant. Complete the following formulas. (You may use  $f'$ ,  $g'$  and  $u'$  as shorthand):

(a)  $\frac{d}{dx}(cf(x)) = \underline{c \cdot f'(x)}$  (b)  $\frac{d}{dx}(f(x) \cdot g(x)) = \underline{f'g + fg'}$  (c)  $\frac{d}{dx}e^u = \underline{u'e^u}$

(d)  $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \underline{\frac{f'g - fg'}{g^2}}$  (e)  $\frac{d}{dx}\ln u = \underline{\frac{u'}{u}}$  (f)  $\frac{d}{dx}f(g(x)) = \underline{f'(g(x)) \cdot g'(x)}$

(g)  $\frac{d}{dx}x^n = \underline{nx^{n-1}}$  (h)  $\frac{d}{dx}a^u = \underline{u'a^u \ln a}$

2. An airplane flying at an altitude of 5 miles passes directly over a radar tower. When the distance between the tower and the airplane is 10 miles, the tower detects that its distance from the plane is changing at a rate of 240 miles per hour. How fast is the plane flying?

The equation I used (before differentiating) is  $\underline{y^2 = x^2 + 5^2}$  (or something similar) 

After differentiating I have  $\underline{2y \frac{dy}{dt} = 2x \frac{dx}{dt}}$

Therefore, the plane is traveling at a speed of  $\underline{480/\sqrt{3}}$  miles per hour.

3. Find  $x$ -values of the critical points (if they exist) of the function  $f(x) = 3x^4 - 6x^2$ . List the  $x$ -values separated by commas below. If there are none, write "none".

Critical points:  $x = \underline{-1, 0, 1}$

4. For the function above, find the absolute extrema on  $[1, 2]$ .

Absolute maximum(s):  $\underline{f(2) = 24}$

Absolute minimum(s):  $\underline{f(1) = -3}$

5. The half-life of an ingredient in Jhevon's hotdogs is 128 days. If the ingredient decays radioactively, answer the following; assuming you have 2 pounds of the ingredient and  $P(t)$  represents the amount of the ingredient at time  $t$ .

(a) Write down the differential equation with initial condition for the "secret" ingredient:

Differential equation:  $\underline{P' = -\frac{\ln 2}{128} P}$  Initial condition:  $\underline{P(0) = 2}$

(b) A formula for  $P(t)$  is:  $P(t) = \underline{P = 2e^{-\frac{\ln 2}{128} t}}$

(c) After how long will there be 0.2 pounds of the ingredient? You may leave  $e$ 's and  $\ln$ 's in your answer:  $t = \underline{\frac{-128 \ln 0.1}{\ln 2}}$

**Bonus:**

1. (3 points) Provide a naive sketch of the graph from problem 3. You need not consider concavity. Plot intercepts and extrema.

