Math 205 Mock Final
July 25, 2016

Name: ____

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Do this exam, without cheating, in 2 hours and 15 minutes.

3. This exam has two parts, all problem in part 1 are compulsory, while you must choose four problems from part 2.

4. Um, that's it. This is a mock exam. Good luck!

Part 1: Do all problems in this part.

1. (4 points each part) Find $\frac{dy}{dx} = y'$ for each of the following:

(a) $y = x\sqrt[3]{\ln x}$

(b) $y = \frac{xe^x}{(x+1)^2}$

(c) $y = \sqrt{x}(e^x + \ln x)^2$

(d) $y = \ln\left[\frac{x^x}{x(x+1)^3(x-7)}\right]$

2. (5 points each) Evaluate the following integrals:

$(a) \quad \int \frac{(x+1)(x^2-3)}{x}\, dx$

$(b)\ \int \frac{5\sqrt{\ln x}}{4x}\,dx$

$(c)\ \int \frac{4}{e^{5x}}\,dx$

$(d)\ \int_0^1 \frac{2x}{\sqrt{x^2+1}}\,dx$

(e) $\displaystyle\int_{1}^{\sqrt[3]{\ln 3}} x^2 e^{x^3}\, dx$

3. (3 points each) Simplify the following:

(a) $e^{\ln e^x + y} - e^{3\ln x}$

(b) $\ln \sqrt[3]{\dfrac{x(x-1)^2 e^2}{\sqrt{x}(x^3+1)^3}}$

4. In 3 days, a 10 gram sample of a radioactive material decays to 3 grams. Let $P(t)$ be the mass remaining after time t in days.
   (a) Find the differential equation satisfied by $P(t)$ and also its initial condition.

   (b) Find $P(t)$ and simplify.

5. (a) State the limit definition to find the derivative $f'(x)$ of a function $f(x)$ and use this definition to find $f'(3)$ for the function $f(x) = -\frac{2}{x-1}$

   (b) Use part (a) to find the tangent line to $y = f(x)$ at the point where $x = 3$.

Part 2: Complete any four problems in this section. Each problem is worth 10 points.

6. For the function $f(x) = \frac{x}{1-x^2}$ find (provided they exist) the domain, intercepts, asymptotes, local extrema, inflection point(s), intervals of increasing and decreasing, and intervals of concavity. You may assume, without verification, that $f'(x) = \frac{x^2+1}{(1-x^2)^2}$ and $f''(x) = \frac{2x(x^2+3)}{(1-x^2)^3}$.

7. (a) A particle is traveling on the curve $y^2 + xy = 2$. As the particle goes through the point (1,1), the x-coordinate is decreasing at a rate of 3 units per second. Find the rate at which the y-coordinate is changing at this moment.

(b) Compute the following limits:

$(i)\ \lim_{x \to 1} \frac{x^2 - 1}{1 - x^4}$

$(ii)\ \lim_{x \to -\infty} \frac{3 - x^{\pi} + 5x^{12}}{4 + 3x^2 - 2x^5 - x^{12}}$

8. (a) An artist is planning to sell signed prints of her latest work. If 50 copies are offered, she can charge \$400 each. But, if she makes more than 50 prints, she must lower the price of all prints by \$5 for each print in excess of the 50. How many prints should the artist make to maximize her revenue?

(b) Unrelated to part (a), suppose a carpenter makes and sells 3000 wooden chairs per year. For each chair, it costs him \$2 per year to store it in his warehouse. Ordering parts to make x new chairs, costs him \$55 per delivery. Find the cost function, $C(x)$, that must be minimized if the carpenter wishes to minimize his inventory cost for the chairs.

9. (a) Use Riemann sums with 4 subintervals and right hand endpoints to estimate the area under $y = x^2$ on $0 \leq x \leq 1$.

(c) Use integration to find the exact area under the curve, which you estimated above.

10. An object is launched from a height of 256 feet with velocity $v(t) = -32t + 96$ after t seconds.

(a) Find the position function, $s(t)$, that gives the height of the particle at time t.

(b) When will the object hit the ground?

(c) How high will the object get?

11. Roughly sketch the curves $y = -x^2 + 6x - 5$ and $y = 2x - 5$ on the same pair of axes, and find the area between the curves.