

MATH 203 QUIZ 7 - Version B

June 26, 2014

Name: ANSWERS

Instructions: (1) No calculators! (2) Use your own scrap paper. (3) Write your answers in the space provided. Assume all functions are differentiable.

1. Suppose $F(x, y, z) = 0$ defines a level surface. Write the formula for the equation of the tangent plane to $F(x, y, z) = 0$ at the point (a, b, c) .

$F_x(x-a) + F_y(y-b) + F_z(z-c) = 0$ where F_x, F_y, F_z are evaluated at (a, b, c)

2. For the function $f(x, y)$, define $\nabla f = \underline{\langle f_x, f_y \rangle}$

3. Using a dot product, define $D_{\vec{u}}f = \underline{\nabla f \cdot \vec{u}}$

4. Let $f(x, y, z) = 4x^2 + 3y^3 - xy + 4z$. (a) Find the directional derivative of f at the point $(1, -1, 2)$ in the direction of the point $(2, 0, 4)$.

(i) $\vec{u} = \underline{\frac{\langle 1, 1, 2 \rangle}{\sqrt{6}}}$ (ii) $D_{\vec{u}}f = \underline{\frac{25}{\sqrt{6}}}$

(b) What is the maximum rate of change at the point $(1, -1, 2)$? $\underline{|\nabla f| = \sqrt{161}}$

(c) Give a unit vector for the direction of the max rate of change. $\underline{\frac{\langle 9, 8, 4 \rangle}{\sqrt{161}}}$

5. (a) Define "D", the formula used to classify the critical points of a function $f(x, y)$.

$D = \underline{f_{xx}f_{yy} - (f_{xy})^2}$

(b) Find and classify the critical points of $f(x, y) = xy(x+y-1)$. (No credit for classification if the wrong critical point is given. So solve for them carefully!)

Critical point 1: $\left(0, 0\right)$ Classification saddle point

Critical point 2: $\left(0, 1\right)$ Classification saddle point

Critical point 3: $\left(1, 0\right)$ Classification saddle point

Critical point 4: $\left(\frac{1}{3}, \frac{1}{3}\right)$ Classification minimum point

Bonus 1: Set up an integral to compute the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

Integral set-up: $\int_0^5 \int_0^4 (16 - x^2) dx dy$ or $\int_0^4 \int_0^5 (16 - x^2) dy dx$ Volume $\underline{\frac{640}{3}}$

Bonus 2: Evaluate $\int_0^1 \int_y^1 e^{y/x} dx dy = \underline{\frac{e-1}{2}}$. Hint: reverse the order of integration.