**Math 203 Quiz 4B**
September 17, 2015

**Name:** ANSWERS

**Instructions: No calculators! Answer all problems in the space provided!**

1. Complete the following statements:
   (a) $\vec{a} \cdot \vec{b} = 0$ iff $\vec{a}$ and $\vec{b}$ are orthogonal (b) $\vec{a} \times \vec{b} = \vec{0}$ iff $\vec{a}$ and $\vec{b}$ are parallel
   (c) $\vec{a} = c\vec{b}$ iff $\vec{a}$ and $\vec{b}$ are parallel

2. If $\theta$ is the angle between $\vec{a}$ and $\vec{b}$, then, in terms of $\theta$:
   (a) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ (b) $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

3. State the required form for the equation of a line (in 3D), and define terms in part (d):
   (a) Vector form: $\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$
   (b) Parametric form: $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$
   (c) Symmetric form: $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
   (d) Define terms/symbols above, i.e., using the same symbols you did above, state a point the line passes through and the direction of the line as a vector:
   (ii) Point: $(x_0, y_0, z_0)$ (ii) direction vector: $\langle a, b, c\rangle$

4. (a) Find the vector *equation* of the line that passes through the point (1, 0, 3) that is orthogonal is the two lines $L_1: x = 4 + k, y = -2 - 2k, z = 1 + 3k$ and $L_2: \frac{y-7}{2} = \frac{z+4}{4}; x = 3.$
   $\langle x, y, z\rangle = \langle 1, 0, 3\rangle + t\langle -14, -4, 2\rangle$
   (b) Find the parametric equations of the line that passes through the points (1,1,4) and (5,0,-2):
   $x = 1 + 4t,\ y = 1 - t,\ z = 4 - 6t$ (many possible forms of answer).
   (c) What is the angle between the lines $L_1$ and $L_2$ in part (a)? (you may leave inverse trig functions in your answer):
   $\theta = \cos^{-1}\left(\frac{\langle 1,-2,3\rangle \cdot \langle 0,2,4\rangle}{|\langle 1,-2,3\rangle||\langle 0,2,4\rangle|}\right) = \cos^{-1}\left(\frac{8}{2\sqrt{5}\sqrt{14}}\right) = \cos^{-1}\left(\frac{4}{\sqrt{5}\sqrt{14}}\right)$
   (d) What is the distance between $L_1$ and $L_2$ in part (a)? $d = \frac{|\langle 1,-9,5\rangle \cdot \langle -14,-4,2\rangle|}{\sqrt{14^2+4^2+2^2}}$

**Bonus Problems:**

1. (a) State the formula for the equation of a plane: $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$
   (b) For the above, what is the: (i) normal vector? $\langle a, b, c\rangle$ (ii) a point on the plane? $(x_0, y_0, z_0)$

2. Find the vector equation of the line that passes through the point (2,3,1) that is orthogonal to the plane $x - 2y + 5z = 7$.
   $\langle x, y, z\rangle = \langle 2, 3, 1\rangle + t\langle 1, -2, 5\rangle$