Math 203 Quiz 4A

September 17, 2015

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. State the required form for the equation of a line (in 3D), and define terms in part (d):
   (a) Vector form: $\langle x,y,z\rangle = \langle x_0,y_0,z_0\rangle + t\langle a,b,c\rangle$

   (b) Parametric form: $x = x_0+at,\ y=y_0+bt,\ z=z_0+ct$

   (c) Symmetric form: $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

   (d) Define terms/symbols above, i.e., using the same symbols you did above, state a point the line passes through and the direction of the line as a vector:

   (i) Point: (x_0, y_0, z_0) (ii) direction vector: $\langle a,b,c\rangle$

2. (a) Find the vector *equation* of the line that passes through the point (1, 0, 3) that is orthogonal is the two lines $L_1\!: x = 4 + k, y = -2 + 3k, z = 1 - k$ and $L_2\!: \frac{x-7}{2} = \frac{z+4}{5}; y = 3$.

   $\langle x,y,z\rangle = \langle 1,0,3\rangle + t\langle 15,-7,-6\rangle$

   (b) Find the parametric equations of the line that passes through the points (1,0,4) and (7,-1,2):

   $x=1+6t,\ y=-t,\ z=4-2t$ (many possible answer forms).

   (c) What is the angle between the lines L_1 and L_2 in part (a)? (you may leave inverse trig functions in your answer):

   $\theta = \cos^{-1}\left(\frac{\langle 1,3,-1\rangle\cdot\langle 2,0,5\rangle}{|\langle 1,3,-1\rangle||\langle 2,0,5\rangle|}\right) = \cos^{-1}\left(\frac{-3}{\sqrt{11}\sqrt{29}}\right)$

   (d) What is the distance between L_1 and L_2 in part (a)? $d = \frac{|\langle -3,-5,3\rangle\cdot\langle 15,-7,-6\rangle|}{\sqrt{15^2+7^2+6^2}}$

3. Complete the following statements:
   (a) $\vec{a}\cdot\vec{b} = 0$ iff $\vec{a}$ and $\vec{b}$ are orthogonal (b) $\vec{a}\times\vec{b} = \vec{0}$ iff $\vec{a}$ and $\vec{b}$ are parallel
   (c) $\vec{a} = c\vec{b}$ iff $\vec{a}$ and $\vec{b}$ are parallel

4. If θ is the angle between $\vec{a}$ and $\vec{b}$, then, in terms of θ:

   (a) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ (b) $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

Bonus Problems:

1. (a) State the formula for the equation of a plane: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

   (b) For the above, what is the: (i) normal vector? $\langle a,b,c\rangle$ (ii) a point on the plane? (x_0,y_0,z_0)

2. Find the vector equation of the line that passes through the point (2,3,1) that is orthogonal to the plane $x - 2y + 5z = 7$.

   $\langle x,y,z\rangle = \langle 2,3,1\rangle + t\langle 1,-2,5\rangle$