Math 203 Quiz 3B
September 8, 2015

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. If θ is the angle between $\vec{a}$ and $\vec{b}$, then, in terms of θ:

   (a) $\vec{a} \cdot \vec{b} =$ $|\vec{a}||\vec{b}|\cos\theta$ (b) $|\vec{a} \times \vec{b}| =$ $|\vec{a}||\vec{b}|\sin\theta$

2. Give the formulas for: (a) $\text{comp}_{\vec{a}}\vec{b} =$ $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ (b) $\text{proj}_{\vec{a}}\vec{b} =$ $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$

3. Let $\vec{a} = < a_1, a_2 >$ and $\vec{b} = < b_1, b_2 >$. What are the formulas for:

   (a) $\vec{a} \cdot \vec{b} =$ $a_1b_1 + a_2b_2$ (b) $|\vec{b}| =$ $\sqrt{b_1^2 + b_2^2}$

   (c) $3\vec{a} =$ $\langle 3a_1, 3a_2 \rangle$ (d) $3\vec{a} - 5\vec{b} =$ $\langle 3a_1 - 5b_1, 3a_2 - 5b_2 \rangle$

4. (a) Compute $< 1, -2, 3 > \times < 0, 2, 4 > =$ $\langle -14, -4, 2 \rangle$

   (b) What is the super special property of your answer to 4(a) in regards to the vectors involved?

   $\langle -14, -4, 2 \rangle$ is orthogonal to both the original vectors

5. Complete the following statements:
   (a) $\vec{a} = c\vec{b}$ iff $\vec{a}$ and $\vec{b}$ are parallel
   (b) $\vec{a} \cdot \vec{b} = 0$ iff $\vec{a}$ and $\vec{b}$ are orthogonal
   (c) $\vec{a} \times \vec{b} = \vec{0}$ iff $\vec{a}$ and $\vec{b}$ are parallel

Bonus Problems:

1. State the required form for the equation of a line (in 3D):
   (a) Parametric form: $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$
   (b) Vector form: $\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$
   (c) Symmetric form: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

2. (a) State the formula for the equation of a plane: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
   (b) Based on the symbols you used above, what is the normal vector? $\vec{n} = \langle a, b, c \rangle$
   (c) What is a point the plane passes through? (x_0, y_0, z_0)

3. A big boat is being pulled by two smaller boats. One of the smaller boats is pulling at an angle of $\frac{\pi}{3}$ to the easterly direction at 3 m/s. The other boat pulls at an angle of $-\frac{\pi}{6}$ to the easterly direction at 2 m/s. In what direction will the boat move? (Assume you have a bird's eye view and the tip of the big boat is your "origin". Give the direction as a vector). $\left\langle \frac{3}{2} + \sqrt{3},\ \frac{3\sqrt{3}}{2} - 1 \right\rangle$