Math 203 Quiz 3A
September 8, 2015

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. Let $\vec{a} = < a_1, a_2 >$ and $\vec{b} = < b_1, b_2 >$. What are the formulas for:

   (a) $\vec{a} \cdot \vec{b} =$ $a_1b_1 + a_2b_2$ (b) $|\vec{a}| =$ $\sqrt{a_1^2 + a_2^2}$

   (c) $2\vec{a} =$ $\langle 2a_1, 2a_2 \rangle$ (d) $2\vec{a} - 3\vec{b} =$ $\langle 2a_1 - 3b_1, 2a_2 - 3b_2 \rangle$

2. (a) Compute $< 1,3,-1 > \times < 2,0,5 > =$ $\langle 15, -7, -6 \rangle$

   (b) What is the super special property of your answer to 2(a) in regards to the vectors involved?

   $\langle 15, -7, -6 \rangle$ is orthogonal to both the original vectors

3. Complete the following statements:
   (a) $\vec{a} \cdot \vec{b} = 0$ iff $\vec{a}$ and $\vec{b}$ are orthogonal
   (b) $\vec{a} \times \vec{b} = \vec{0}$ iff $\vec{a}$ and $\vec{b}$ are parallel
   (c) $\vec{a} = c\vec{b}$ iff $\vec{a}$ and $\vec{b}$ are parallel

4. If θ is the angle between $\vec{a}$ and $\vec{b}$, then, in terms of θ:

   (a) $\vec{a} \cdot \vec{b} =$ $|\vec{a}||\vec{b}| \cos\theta$ (b) $|\vec{a} \times \vec{b}| =$ $|\vec{a}||\vec{b}| \sin\theta$

5. Give the formulas for:

   (a) $\text{comp}_{\vec{a}}\vec{b} =$ $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$ (b) $\text{proj}_{\vec{a}}\vec{b} =$ $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\vec{a}$

Bonus Problems:

1. State the required form for the equation of a line (in 3D):
   (a) Vector form: $\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$
   (b) Parametric form: $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$
   (c) Symmetric form: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

2. (a) State the formula for the equation of a plane: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
   (b) Based on the symbols you used above, what is the normal vector? $\vec{n} = \langle a, b, c \rangle$
   (c) What is a point the plane passes through? (x_0, y_0, z_0)

3. A big boat is being pulled by two smaller boats. One of the smaller boats is pulling at an angle of $\frac{\pi}{6}$ to the easterly direction at 4 m/s. The other boat pulls at an angle of $-\frac{\pi}{4}$ to the easterly direction at 1 m/s. In what direction will the boat move? (Assume you have a bird's eye view and the tip of the big boat is your "origin". Give the direction as a vector). $\left\langle 2\sqrt{3} + \frac{\sqrt{2}}{2},\ 2 - \frac{\sqrt{2}}{2} \right\rangle$