Math 203 Quiz 5B
September 29, 2015

Name: ANSWERS

Instructions: No calculators! Answer <u>all</u> problems in the space provided!

1. Let $\vec{r}(t) = < x(t), y(t) >$. What is:

   (a) $\lim\limits_{t\to a} \vec{r}(t) =$ $\left\langle \lim\limits_{t\to a} x(t), \lim\limits_{t\to a} y(t) \right\rangle$ (b) $\vec{r}\,'(t) =$ $\langle x'(t), y'(t) \rangle$

2. Give the formula for the equation of the plane and the meaning of the symbols used:

   Formula: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ Meanings: $\langle a,b,c\rangle$ = normal vector
   (x_0, y_0, z_0) = point in plane

3. Let $\vec{r}(t) = \langle \frac{1}{t-3}, te^t, \ln(t)+1 \rangle$. (a) What is the domain of $\vec{r}(t)$? $(0,3)\cup(3,\infty)$

   (b) Compute $\vec{r}\,'(t) =$ $\left\langle -\frac{1}{(t-3)^2}, te^t+e^t, \frac{1}{t} \right\rangle$

   (c) Compute $\int \vec{r}(t)\,dt =$ $\langle \ln|t-3|, te^t-e^t, t\ln t \rangle + \vec{C}$

   (d) Compute $\lim\limits_{t\to e^5} \vec{r}(t) =$ $\left\langle \frac{1}{e^5-1}, e^5e^{e^5}, 6 \right\rangle$

4. Find the *equation* of the tangent line to $\vec{r}(t)$ above at $(-1/2, e, 1)$: $\langle x,y,z\rangle = \langle -\tfrac{1}{2}, e, 1\rangle + t\langle -\tfrac{1}{4}, 2e, 1\rangle$

5. Find an equation for the plane that passes through $(-3,4,-1)$ that contains the line $\vec{r}(t) =< 2,-3,1 > +t < -1,2,4 >$

   $32(x+3)+22(y-4)-3(z+1)=0$

6. Find the equation of the line through $(1,2,3)$ that is orthogonal to the plane $7x-2y-3z=7$

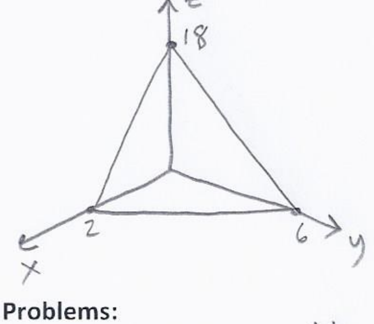
   $\langle x,y,z\rangle = \langle 1,2,3\rangle + t\langle 7,-2,-3\rangle$

7. Find the point of intersection of the line $x=1+4t, y=2t, z=3+6t$ and the plane $2x+3y=-5$: $(-1,-1,0)$

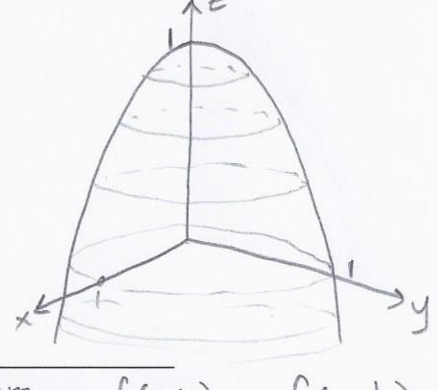
8. Give the formula for the unit tangent vector of a function $\vec{r}(t)$: $\vec{T} = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

9. Sketch the graph of the given function:

   (a) $f(x,y) = 18-9x-3y$



   (b) $f(x,y) = 1-x^2-y^2$



Bonus Problems:

1. Find $\lim\limits_{(x,y)\to(2,-2)} \frac{x^2-y^2}{x+y} =$ 4
2. Define "$f(x,y)$ is continuous at (a,b)" using an equation: $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$
3. Using limits, define $\frac{\partial f}{\partial y} = f_y =$ $\lim\limits_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$