Math 203 Quiz 5A

September 29, 2015

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. Give the formula for the equation of the plane and the meaning of the symbols used:
   Formula: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ Meanings: $\langle a,b,c\rangle$ = normal vector
   $\langle x_0,y_0,z_0\rangle$ = point in the plane

2. Let $\vec{r}(t) = < x(t), y(t) >$. What is:

   (a) $\lim_{t\to a} \vec{r}(t) = \langle \lim_{t\to a} x(t), \lim_{t\to a} y(t)\rangle$ (b) $\vec{r}'(t) = \langle x'(t), y'(t)\rangle$

3. Let $\vec{r}(t) = \langle te^t, \frac{1}{t-2}, \ln(t) - 5\rangle$. (a) What is the domain of $\vec{r}(t)$? $(0,2)\cup(2,\infty)$

   (b) Compute $\vec{r}'(t) = \langle te^t + e^t, -\frac{1}{(t-2)^2}, \frac{1}{t}\rangle$

   (c) Compute $\int \vec{r}(t)\,dt = \langle te^t - e^t, \ln|t-2|, t\ln t - 6t\rangle + \vec{C}$

   (d) Compute $\lim_{t\to e^5} \vec{r}(t) = \langle e^5 e^{e^5}, \frac{1}{e^5-2}, 0\rangle$

4. Find the *equation* of the tangent line to $\vec{r}(t)$ above at $(e,-1,-5)$: $\langle x,y,z\rangle = \langle e,-1,-5\rangle + t\langle 2e,-1,1\rangle$

5. Find an equation for the plane that passes through $(2,3,-1)$ that contains the line $\vec{r}(t) =< 3,-1,0 > +t < 5,-1,1 >$

   $3(x-2)-4(y-3)-19(z+1)=0$

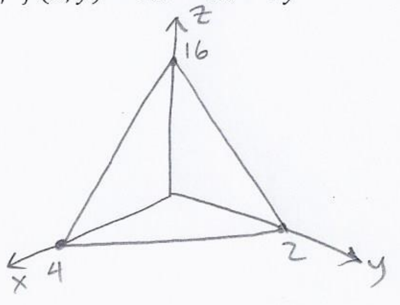
6. Find the equation of the line through $(1,2,3)$ that is orthogonal to the plane $2x - 3y + 5z = 5$

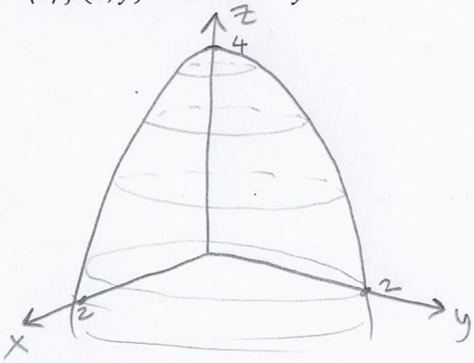
   $\langle x,y,z\rangle = \langle 1,2,3\rangle + t\langle 2,-3,5\rangle$

7. Find the point of intersection of the line $x = 1 + 3t, y = -2t, z = 1 + t$ and the plane $x - y + z = 6$: $(3, -\frac{4}{3}, \frac{5}{3})$

8. Give the formula for the unit tangent vector of a function $\vec{r}(t)$: $\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$

9. Sketch the graph of the given function:
   (a) $f(x,y) = 16 - 4x - 8y$ (b) $f(x,y) = 4 - x^2 - y^2$





Bonus Problems:

1. Find $\lim_{(x,y)\to(3,-3)} \frac{x^2-y^2}{x+y} =$ 6
2. Define "$f(x,y)$ is continuous at (a,b)" using an equation: $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$
3. Using limits, define $\frac{\partial f}{\partial x} = f_x = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$