Math 203 Test 4A
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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

They survived to the end of calc 3??

Must state what test is being used!

1. (10 points each) For each of the following series, state, with justification, whether the series converges absolutely, converges conditionally, or diverges.

$$(a)\ \sum_{n=1}^{\infty}(-1)^n \cos\left(\frac{10^n}{n!}\right)$$

$\lim_{n\to\infty} \cos\frac{10^n}{n!} = \cos 0 = 1 \neq 0$

$\Rightarrow$ The series diverges by test for divergence.

$$(b)\ \sum_{n=3}^{\infty}\frac{(-1)^n}{n\sqrt{\ln n}}$$

Check abs

$\sum_3^\infty \left|\frac{(-1)^n}{n\sqrt{\ln n}}\right| = \sum_3^\infty \frac{1}{n\sqrt{\ln n}}$

We employ the integral test: $\int_3^\infty \frac{1}{x\sqrt{\ln x}}dx = 2\sqrt{\ln x}\Big|_3^\infty$
(since the func. is continuous, non-negative and decreasing)

$= \infty$

diverges!

Check conditional

Since $a_n = \frac{1}{n\sqrt{\ln n}}$ is decreasing and $\lim a_n = 0$

The series converges conditionally by the Alternating series test

(c) $\sum_{n=0}^{\infty} (-1)^n \frac{2n+1}{\sqrt{4n^3+1}} \sim \frac{2n}{\sqrt{4n^3}} = \frac{2n}{2n^{3/2}} = \frac{1}{n^{1/2}}$

Check abs

$\sum_{0}^{\infty} \left| (-1)^n \frac{2n+1}{\sqrt{4n^3+1}} \right| = \sum_{0}^{\infty} \frac{2n+1}{\sqrt{4n^3+1}}$. Set $a_n = \frac{2n+1}{\sqrt{4n^3+1}}$, $b_n = \frac{1}{n^{1/2}}$

We employ the limit comparison test: $\lim \frac{a_n}{b_n} = 1 > 0 \Rightarrow \sum a_n$ diverges!

Check conditional

Since $a_n = \frac{2n+1}{\sqrt{4n^3+1}}$ is eventually decreasing and $\lim a_n = 0$

The series converges conditionally by the Alternating series test.

(d) $\sum_{n=1}^{\infty} \cos(n\pi) \frac{5\sqrt{n}}{4n^4}$

Check abs

$\sum_{1}^{\infty} \left| \cos(n\pi) \frac{5\sqrt{n}}{4n^4} \right| \leq \sum_{1}^{\infty} \frac{5\sqrt{n}}{4n^4} = \frac{5}{4} \sum_{1}^{\infty} \frac{1}{n^{7/2}} \rightarrow$ convergent p-series.

$\Rightarrow$ The series is absolutely convergent by the comparison test.

(You could also notice that $\cos n\pi = (-1)^n$, $n \geq 1$.
But this would not change the solution.)

(e) $\sum_{n=0}^{\infty} \frac{(-3)^n + 1}{2^{2n}} = \sum_{0}^{\infty} \left(\frac{-3}{4}\right)^n + \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$

$\downarrow$ convergent geometric series $r = -3/4$

$\downarrow$ convergent geometric series $r = 1/4$

$\downarrow$ the sum of two convergent series is a convergent series.

$\downarrow$ geo. series converge abs when convergent

$\Rightarrow$ $\boxed{\text{The series converges absolutely by geometric series test.}}$

(Note $\sum |a_n + b_n| \neq \sum |a_n| + \sum |b_n|$, but $\sum |a_n + b_n| \leq \sum |a_n| + \sum |b_n|$ so we could use comparison w/ abs conv. test also.)

(f) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)^n$

$$\lim \left(\frac{n}{n+1}\right)^n = \lim \frac{1}{\left(\frac{n+1}{n}\right)^n}$$

$$= \lim \frac{1}{\left(1 + \frac{1}{n}\right)^n}$$

$$= \frac{1}{e}$$

$$\neq 0$$

$\boxed{\text{The series diverges by the test for divergence.}}$

(g) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

Check abs

$\sum_1^{\infty} \left| \frac{(-1)^n}{\ln(n+1)} \right| = \sum_1^{\infty} \frac{1}{\ln(n+1)}$. Note that $\sum_1^{\infty} \frac{1}{n} \leq \sum_1^{\infty} \frac{1}{\ln(n+1)}$

and $\sum_1^{\infty} \frac{1}{n}$ diverges $\Rightarrow$ the series diverges.

Check conditional

Since $a_n = \frac{1}{\ln(n+1)}$ is decreasing and $\lim a_n = 0$,

The series converges conditionally by the Alternating series test

(h) $\sum_{n=0}^{\infty} \frac{(-1)^n \cos n^2}{n^2+1}$

Check abs

$\sum_0^{\infty} \left| \frac{(-1)^n \cos n^2}{n^2+1} \right| \leq \sum_0^{\infty} \frac{1}{n^2+1} \leq 1 + \underbrace{\sum_1^{\infty} \frac{1}{n^2}}_{\text{convergent p-series.}}$

$\Rightarrow$ The series is absolutely convergent by the comparison test.