Math 203 Quiz 9B

November 3, 2015

Name: ANSWERS

Instructions: No calculators! Answer <u>all</u> problems in the space provided!

1. Suppose you can describe a region D by $D = \{(x,y)|\ c \le y \le d, h_1(y) \le x \le h_2(y)\}$ and assume you can easily integrate $f(x,y)$ with respect to x or y. How would you set up?

   $\iint_D f(x,y)dA =$ $\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$

2. Suppose you have a rectangular region $R = [a,b] \times [c,d]$ and it is difficult to integrate $f(x,y)$ with respect to y. How would you set up?

   $\iint_R f(x,y)dA =$ $\int_c^d \int_a^b f(x,y)\,dx\,dy$

3. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [0,1] \times [1,2]$.

   Integral set-up: $\int_0^1 \int_1^2 3y^2 - x^2 + 2\,dy\,dx$ or $\int_1^2 \int_0^1 3y^2 - x^2 + 2\,dx\,dy$ Volume: $26/3$

4. Find the volume of the solid under the surface $z = 2 + x^2y^2$ that lies above the region enclosed by $x = y^2$ and $x = 4$.

   Integral set-up: $\int_{-2}^{2} \int_{y^2}^{4} 2 + x^2y^2\,dx\,dy$ or $\int_0^4 \int_{-\sqrt{x}}^{\sqrt{x}} 2 + x^2y^2\,dy\,dx$ Volume: $\frac{2624}{27}$

5. Evaluate the following integrals:

   (a) $\int_0^1 \int_y^1 e^{y/x}\,dx\,dy =$ $\frac{1}{2}(e-1)$ (b) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2)\,dx\,dy =$ 1

Bonus Problems:

1. Use a double integral to compute the area of the region in the xy-plane enclosed by $y = x$ and $y = x^2$.

   Integral set-up: $\int_0^1 \int_{x^2}^{x} 1\,dy\,dx$ Area: $\frac{1}{6}$

2. Use polar coordinates to set-up and evaluate the integral

$$\iint_D \sqrt{x^2 + y^2 + 1}\,dA$$

where D is the region in the first quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Integral set-up: $\int_0^{\frac{\pi}{2}} \int_1^2 \sqrt{r^2+1} \cdot r\,dr\,d\theta$ Answer: $\frac{\pi}{6}(5\sqrt{5} - 2\sqrt{2})$