Math 203 Quiz 9A
November 3, 2015

Name: ANSWERS

Instructions: No calculators! Answer <u>all</u> problems in the space provided!

1. Suppose you can describe a region D by $D = \{(x,y) |\ a \le x \le b, g_1(x) \le y \le g_2(x)\}$ and assume you can easily integrate $f(x,y)$ with respect to x or y. How would you set up?

$$\iint_D f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

2. Suppose you have a rectangular region $R = [a,b] \times [c,d]$ and it is difficult to integrate $f(x,y)$ with respect to x. How would you set up?

$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

3. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1,1] \times [1,2]$.

Integral set-up: $\int_{-1}^{1}\int_{1}^{2} 3y^2 - x^2 + 2\,dy\,dx$ or $\int_{1}^{2}\int_{-1}^{1} 3y^2 - x^2 + 2\,dx\,dy$ Volume: $52/3$

4. Find the volume of the solid under the surface $z = 1 + x^2y^2$ that lies above the region enclosed by $x = y^2$ and $x = 4$.

Integral set-up: $\int_{-2}^{2}\int_{y^2}^{4} 1 + x^2y^2\,dx\,dy$ or $\int_{0}^{4}\int_{-\sqrt{x}}^{\sqrt{x}} 1 + x^2y^2\,dy\,dx$ Volume: $\frac{2336}{27}$

5. Evaluate the following integrals:

$$(a)\ \int_0^1 \int_x^1 e^{x/y}\,dy\,dx = \frac{1}{2}(e-1) \qquad (b)\ \int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\,dx\,dy = 0$$

Bonus Problems:

1. Use a double integral to compute the area of the region in the xy-plane enclosed by $y = \sqrt{x}$ and $y = x^2$.

Integral set-up: $\int_0^1 \int_{x^2}^{\sqrt{x}} 1\,dy\,dx$ Area: $1/3$

2. Use polar coordinates to set-up and evaluate the integral

$$\iint_D \sqrt{x^2 + y^2 + 1}\,dA$$

where D is the region in the first quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

Integral set-up: $\int_0^{\pi/2} \int_1^3 \sqrt{r^2+1} \cdot r\,dr\,d\theta$ Answer: $\frac{2}{3}\sqrt{63 - 5\sqrt{5}}\,\pi$