Math 203 Quiz 7A

October 20, 2015

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. Suppose $w = f(x,y,z)$ and $x = x(q,r,s)$, $y = y(q,r,s)$ and $z = z(q,r,s)$. Write down a formula for,

   $\frac{\partial w}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$ or $f_x x_r + f_y y_r + f_z z_r$ (may use "w" instead of "f")

2. Find the indicated derivative for the given function.

   (a) $z = x\ln(x+2y)$, $x = \sin t$, $y = \cos t$. $\frac{\partial z}{\partial t} = \left(\ln(x+2y) + \frac{x}{x+2y}\right)\cos t - \frac{2x}{x+2y}\sin t$

   (b) $z = \frac{2x}{y}$, $x = se^{-t}$, $y = 1 + se^{t}$. $\frac{\partial z}{\partial s} = \frac{2e^{-t}}{y} - \frac{2x}{y^2}e^{t}$

   (c) For the above problem, find $z_t(2,3) = \frac{2}{3} - \frac{8}{9} = -2/9$

3. Suppose $W(s,t) = F(u(s,t), v(s,t))$. Also, $u(1,0) = 2$, $u_s(1,0) = -2$, $u_t(1,0) = 6$, $v(1,0) = 3$, $v_s(1,0) = 5$, $v_t(1,0) = 4$, $F_u(2,3) = -1$ and $F_v(2,3) = 10$. Find $W_s(1,0)$.

   $W_s(1,0) =$ 52

4. A function $z = f(x,y)$ is defined implicitly by $xyz = \cos(x+y+z) - \ln\left(\frac{xy}{z}\right)$. What is,

   $\frac{\partial z}{\partial x} = -\frac{yz + \sin(x+y+z) + 1/x}{xy + \sin(x+y+z) - 1/z}$

5. Suppose $f = f(x,y,z)$. Define $\nabla f = \langle f_x, f_y, f_z \rangle$ or $\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

6. Using a dot product, define $D_{\vec{u}}f = \nabla f \cdot \vec{u}$

7. Suppose $F(x,y,z) = 0$ defines a level surface. Write down an equation for the tangent plane to $F(x,y,z) = 0$ at the point (a,b,c). $F_x(x-a) + F_y(y-b) + F_z(z-c) = 0$ (F_x, F_y, F_z are evaluated at (a,b,c))

8. Let $f(x,y,z) = 3x^2 + 4y^3 - 2xy + 4z$. (a) Find the directional derivative of f at the point (1,-1,2) in the direction of the point (2,0,4).

   (i) $\vec{u} = \frac{\langle 1,1,2 \rangle}{\sqrt{6}}$ (ii) $D_{\vec{u}}f = 26/\sqrt{6}$

   (b) What is the maximum rate of change at the point (1,-1,2)? $|\nabla f| = 6\sqrt{5}$

   (c) Give the unit vector for the direction of max rate of change. $\left\langle \frac{8}{6\sqrt{5}}, \frac{10}{6\sqrt{5}}, \frac{4}{6\sqrt{5}} \right\rangle = \left\langle \frac{4}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}, \frac{2}{3\sqrt{5}} \right\rangle$

Bonus Problems:

1. Let $\vec{u} = \langle a, b \rangle$ be a unit vector. Using limits, define $D_{\vec{u}}f = \lim_{h \to 0} \frac{f(x+ha, y+hb) - f(x,y)}{h}$

2. Define "D", the formula used to classify the critical points of a function $f(x,y)$ in the two-variable second derivative test. $D = f_{xx}f_{yy} - (f_{xy})^2$