Math 203 Quiz 6A

October 13, 2015

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. For a function $f(x, y)$, define, using limits, $\frac{\partial f}{\partial x} = f_x =$ $\lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$

2. For a function $f(x, y)$, write the formula for the tangent plane at (x_0, y_0). You may use "f_x" instead of "$f_x(x_0, y_0)$" and "f_y" instead of "$f_y(x_0, y_0)$".

   $z - z_0 = f_x(x - x_0) + f_y(y - y_0)$

3. Find the equation of the tangent plane to the function $f(x, y) = 3x + 2x^2y - 4xy^3$ at the point where $x = 2$ and $y = 1$.

   $z - 6 = 7(x - 2) - 16(y - 1)$

4. Find the indicated partial derivatives of $f(x, y, z) = x^y + ze^{-x}\cos y$

   (a) $f_x = yx^{y-1} - ze^{-x}\cos y$

   (b) $f_y = x^y \ln x - ze^{-x}\sin y$

   (c) $f_{xx} = -e^{-x}\cos y$

   (d) $\frac{\partial^2 f}{\partial x \partial y} = yx^{y-1}\ln x + x^{y-1} + ze^{-x}\sin y$ ($\frac{x^y}{x}$ acceptable.)

5. What is the formula for the linearization of $f(x, y)$ at the point (a, b)? You may use the shorthand described in 2.

   $L(x, y) = f(a,b) + f_x(x-a) + f_y(y-b)$

6. Let $f(x, y) = \sqrt{x^2 + y^2}$. Use linearization (or differentials) to approximate $f(4.1, 2.9)$.

   $f(4.1, 2.9) \approx 5 + \frac{1}{50} = \frac{251}{50}$

7. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10cm high and 4cm in diameter if the metal in the top and bottom is 0.1cm thick and the metal in the sides is 0.05 cm thick.

   $\frac{14\pi}{5}$ cm^3

Bonus Problems:

1. Suppose $z = f(x, y)$, $x = x(s, t)$ and $y = y(s, t)$: $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$ (could use "z" instead of "f")

2. Let $F(x, y, z) = 0$ be an implicitly defined function. $\frac{dz}{dx} = -\frac{F_x}{F_z}$