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Name: ANSWERS

Instructions: (1) No calculators! (2) Use your own scrap paper. Write your answers in the space provided.

1. Let  $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ . What is

(a)  $\lim_{t \rightarrow a} \mathbf{r}(t) = \langle \lim_{t \rightarrow a} x(t), \lim_{t \rightarrow a} y(t) \rangle$  (b)  $\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$

2. Give the formula for the equation of a plane, and the meaning of the symbols used:

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$  Meanings  $(x_0, y_0, z_0)$  - point on plane  
 $\vec{n} = \langle a, b, c \rangle$  - normal vector.

3. Let  $\mathbf{r}(t) = \left\langle \frac{1}{t-3}, te^t, \ln t + 1 \right\rangle$ . (a) What is the domain of  $\mathbf{r}(t)$ ?  $t \in (0, 3) \cup (3, \infty)$

(b) Compute  $\mathbf{r}'(t) = \left\langle -\frac{1}{(t-3)^2}, te^t + e^t, \frac{1}{t} \right\rangle$

(c) Compute  $\int \mathbf{r}(t) dt = \left\langle \ln|t-3|, te^t - e^t, t \ln t \right\rangle + \vec{C}$

(d) Compute  $\lim_{t \rightarrow e^3} \mathbf{r}(t) = \left\langle \frac{1}{e^3-3}, e^3 e^3, 6 \right\rangle$

4. Find an equation for the plane that passes through  $(-3, 4, -1)$  that contains the line  $L: x = 2 - t, y = -3 + 2t, z = 1 + 4t$

$32(x+3) + 22(y-4) - 3(z+1) = 0$

5. Find the equation of the line through  $(1, 2, 3)$  that is orthogonal to the plane  $7x - 2y - 3z = 7$

$\langle x, y, z \rangle = \langle 1, 2, 3 \rangle + t \langle 7, -2, -3 \rangle$

6. Find the point of intersection of the line  $\langle x, y, z \rangle = \langle 1, 0, 3 \rangle + t \langle 4, 2, 6 \rangle$  and the plane  $2x + 3y = -5$

$(-1, -1, 0)$

7. Give the formula for the unit tangent vector for a function  $\mathbf{r}(t)$

$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$

8. Find the equation of the tangent line to the curve  $\mathbf{r}(t)$  in problem 3. at the point  $(-1/2, e, 1)$

$\langle x, y, z \rangle = \langle -1/2, e, 1 \rangle + t \langle -1/4, 2e, 1 \rangle$

Bonus: Sketch the level curves of  $z^2 = x^2 + y^2$  in the  $xy$ -plane:

