

Name: ANSWERS

Instructions: (1) No calculators! (2) Use your own scrap paper. Write your answers in the space provided.

1. Give the formula for the equation of a plane, and the meaning of the symbols used:

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ Meanings (x_0, y_0, z_0) - point on plane,
 $\vec{n} = \langle a, b, c \rangle$ - normal vector

2. Let $\mathbf{r}(t) = \langle x(t), y(t) \rangle$. What is

(a) $\lim_{t \rightarrow a} \mathbf{r}(t) = \langle \lim_{t \rightarrow a} x(t), \lim_{t \rightarrow a} y(t) \rangle$ (b) $\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$

3. Let $\mathbf{r}(t) = \langle te^t, \frac{1}{t-2}, \ln t - 5 \rangle$. (a) What is the domain of $\mathbf{r}(t)$? $t \in (0, 2) \cup (2, \infty)$

(b) Compute $\mathbf{r}'(t) = \langle te^t + e^t, -\frac{1}{(t-2)^2}, \frac{1}{t} \rangle$

(c) Compute $\int \mathbf{r}(t) dt = \langle te^t - e^t, \ln|t-2|, t \ln t - 6t \rangle + \vec{C}$

(d) Compute $\lim_{t \rightarrow e^2} \mathbf{r}(t) = \langle e^5 e^{e^5}, \frac{1}{e^5-2}, 0 \rangle$

4. Find an equation for the plane that passes through (2,3,-1) that contains the line $\langle x, y, z \rangle = \langle 3, -1, 0 \rangle + t \langle 5, -1, 1 \rangle$

$3(x-2) - 4(y-3) - 19(z+1) = 0$

5. Find the equation of the line through (1,2,3) that is orthogonal to the plane $2x - 3y + 5z = 5$

$\langle x, y, z \rangle = \langle 1, 2, 3 \rangle + t \langle 2, -3, 5 \rangle$

6. Find the point of intersection of the line $L: x = 1 + 3t, y = -2t, z = 1 + t$ and the plane $x + y + z = 6$

$\langle 7, -4, 3 \rangle$

7. Give the formula for the unit tangent vector for a function $\mathbf{r}(t)$

$\vec{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

8. Find the equation of the tangent line to the curve $\mathbf{r}(t)$ in problem 3. at the point $(e, -1, -5)$

$\langle x, y, z \rangle = \langle e, -1, -5 \rangle + t \langle 2e, -1, 1 \rangle$

Bonus: Sketch the level curves of $z = x^2 + y^2$ in the xy -plane:

