Math 202 Test 4A
December 10, 2015

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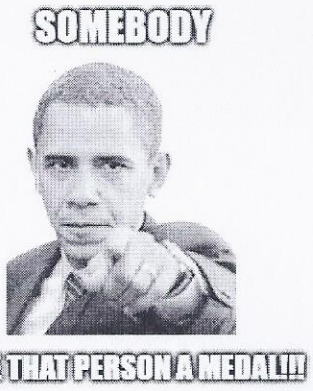
Note that both sides of each page may have printed material.

Instructions:

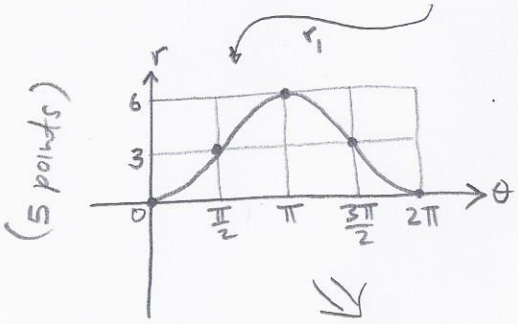
1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

They survived to the last day of calc 2??



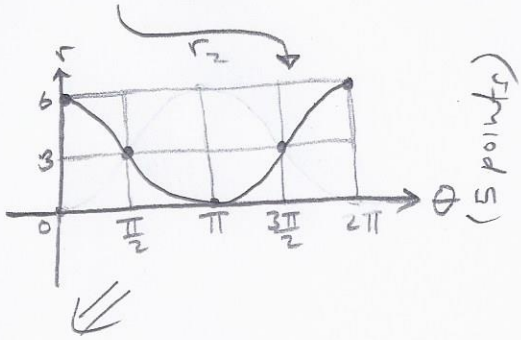
1. (40 points) Sketch both polar graphs on one pair of axes, and find the area outside the first but inside the second.

$r = 3 - 3\cos\theta$

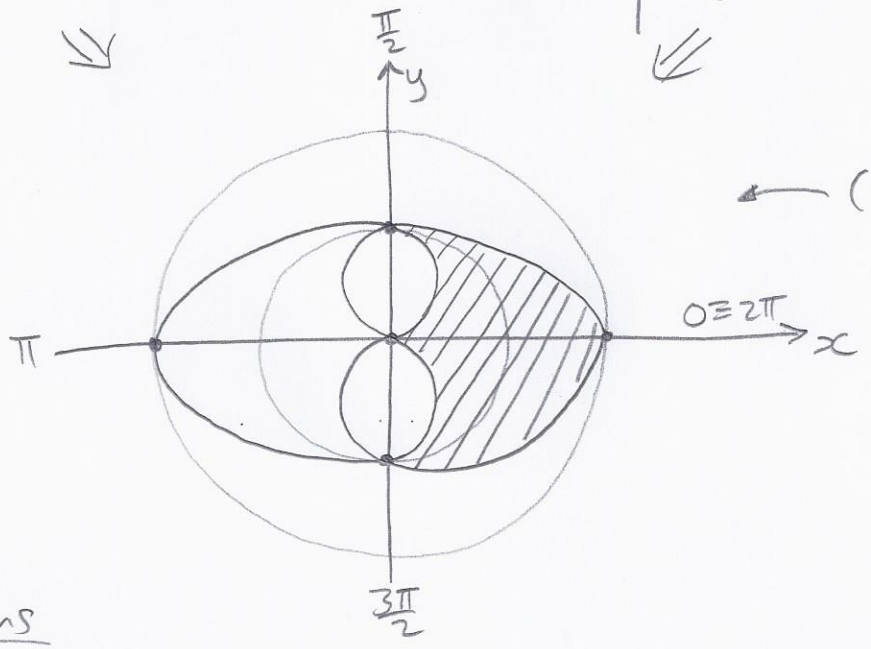


(5 points)

$r = 3 + 3\cos\theta$



(5 points)



← (10 pts for diagram. This includes 2 pts for shading the right area).

Intersections

$3 - 3\cos\theta = 3 + 3\cos\theta$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \pm\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$$\Rightarrow A = \frac{1}{2}\int R^2 - r^2$$

→ 10 pts to set up integral.

$$= 2 \cdot \frac{1}{2}\int_0^{\pi/2} (3+3\cos\theta)^2 - (3-3\cos\theta)^2 \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} 9 + 18\cos\theta + 9\cos^2\theta - 9 + 18\cos\theta - 9\cos^2\theta$$

$$= 36\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$$

$$= 36 \cdot \sin\theta \Big|_0^{\pi/2}$$

→ 8 pts to evaluate integral

$$= \boxed{36}$$

→ 2 pts for correct final answer.

2. (15 points) Set up an integral for the arc length of the polar graph $r = 4\sin 3\theta$ for $0 \le \theta \le \frac{\pi}{4}$.

$$L = \int_0^{\frac{\pi}{4}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad \text{(5 points) formula}$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{(4\sin 3\theta)^2 + (12\cos 3\theta)^2}\, d\theta \quad \text{(5 points) set up}$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{4^2\sin^2 3\theta + 4^2\cos^2 3\theta + 128\cos^2 3\theta}\, d\theta$$

$$= \boxed{\int_0^{\frac{\pi}{4}} \sqrt{16 + 128\cos^2 3\theta}\, d\theta}$$

or

$$\boxed{\int_0^{\frac{\pi}{4}} \sqrt{16\sin^2 3\theta + 144\cos^2 3\theta}\, d\theta}$$

(5 points) simplify

3. (15 points) Find the angle θ needed to eliminate the xy-term in

$$6x^2 + 4\sqrt{3}xy + 2y^2 - 9x + 9\sqrt{3}y - 63 = 0$$

$\Rightarrow A = 6,\ B = 4\sqrt{3},\ C = 2$ → 5 pts for identifying A, B, C.

$$\cot 2\theta = \frac{A - C}{B} \longrightarrow \text{5 pts for set up.}$$

$$= \frac{4}{4\sqrt{3}}$$

$$\Rightarrow \tan 2\theta = \sqrt{3}$$

$$\Rightarrow 2\theta = \frac{\pi}{3}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{6}} \longrightarrow \text{5 pts to evaluate } \theta \text{ correctly.}$$

4. (30 points) Identify the conic section whose equation is $16x^2 - 9y^2 - 32x - 128 = 0$. Make a sketch, and label any of the following which may be present: vertices, foci, asymptotes.

$16x^2 - 32x - 9y^2 - 128 = 0$

$\Rightarrow 16(x^2 - 2x) - 9y^2 = 128$

$\Rightarrow 16[x^2 - 2x + 1 - 1] - 9y^2 = 128$

$\Rightarrow 16[(x-1)^2 - 1] - 9y^2 = 128$

$\Rightarrow 16(x-1)^2 - 16 - 9y^2 = 128$

$\Rightarrow 16(x-1)^2 - 9y^2 = 144$

$\Rightarrow \frac{(x-1)^2}{9} - \frac{y^2}{16} = 1 \longrightarrow$ Hyperbola!

(10 pts to get to formula)

(10 pts for figuring these out)

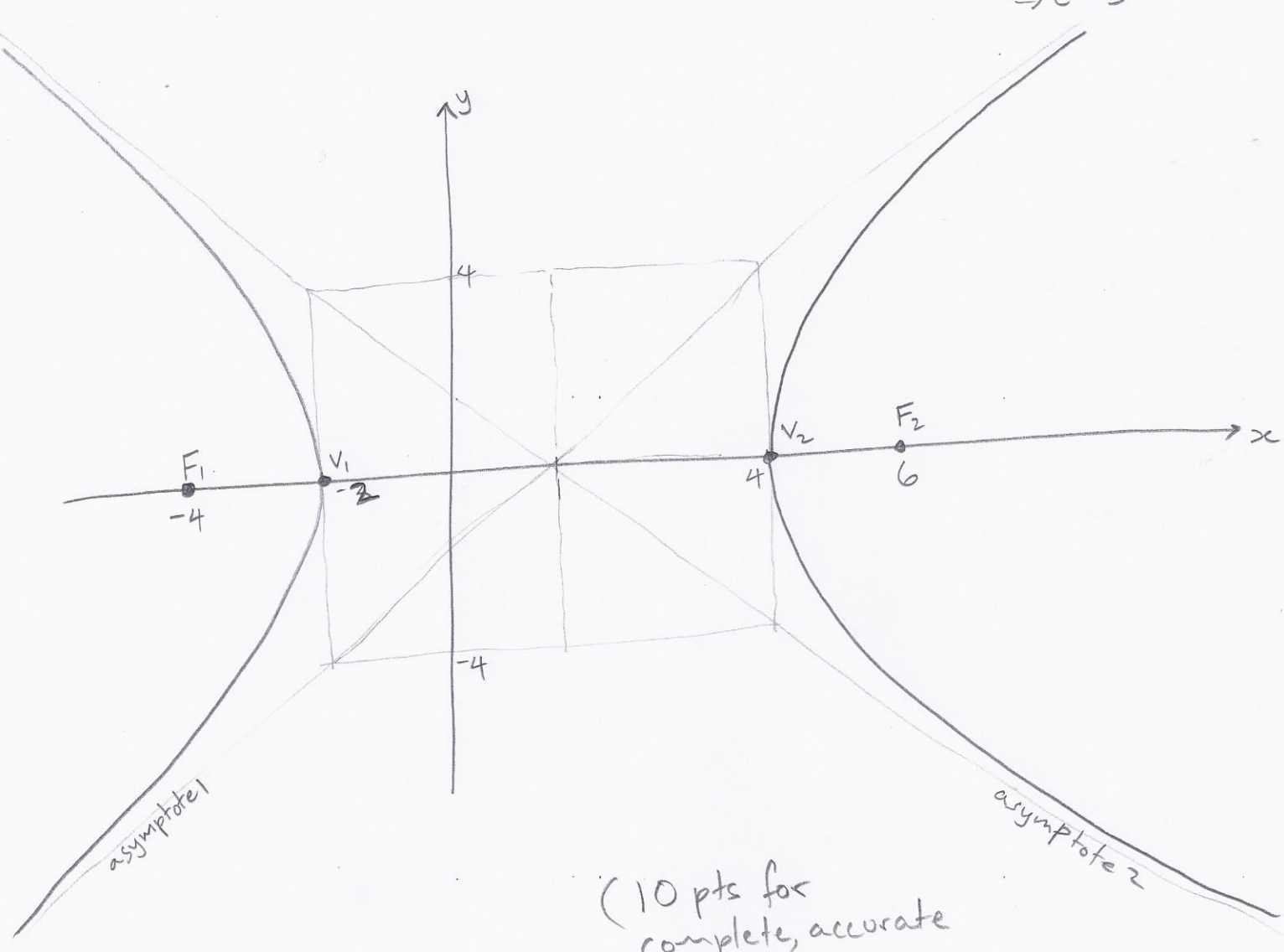
Center $(1, 0)$

Vertices $(1 \pm 3, 0)$

Foci $(1 \pm 5, 0)$

$c^2 = a^2 + b^2 = 9 + 16$

$\Rightarrow c = 5$



(10 pts for complete, accurate sketch)

Bonus:

1. (5 points – all or nothing) What is

$$\int \sec^3 x\, dx =$$

$\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

(all or nothing)

2. (5 points) Evaluate:

$$\int_0^1 \int_1^2 xy^2\, dy\, dx$$

$$= \int_0^1 \frac{xy^3}{3}\Big|_1^2 dx$$

$$= \int_0^1 \frac{8}{3}x - \frac{1}{3}x\, dx$$

$$= \frac{7}{3}\int_0^1 x\, dx$$

$$= \frac{7}{3}\frac{x^2}{2}\Big|_0^1$$

$$= \boxed{\frac{7}{6}}$$

3. (3 points) Complete the table:

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin\theta$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
$\cos\theta$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$

(1 pt per correct column)

4. (1 point) State the integration by parts formula: $\int u\,dv = uv - \int v\,du$

5. (1 point) What mnemonic is used to choose the parts? LIATE

(all or nothing)